

## MAPPING $b$ VALUE IN THE MARMARA REGION (NW TURKEY): IMPLICATIONS FOR LOCAL RECURRENCE TIMES AND SEISMIC HAZARD

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**Key Words:**  $b$  value;  $T_L$  value; North Anatolian Fault Zone; Seismic hazard; Turkey.

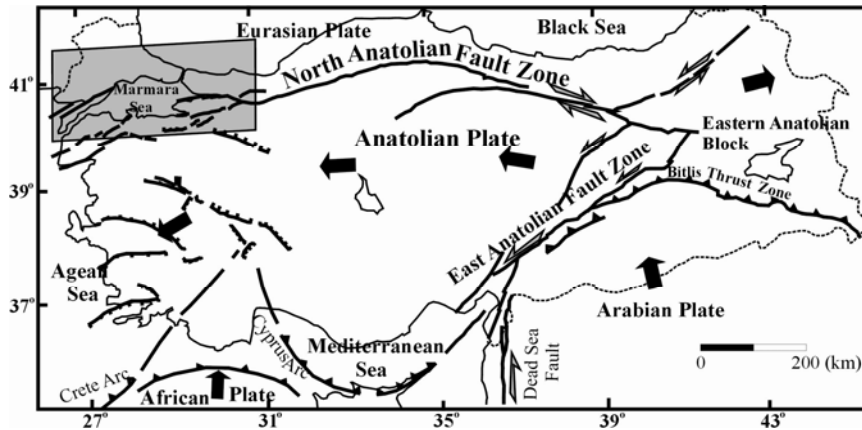
**ABSTRACT** In the present study, we map the  $b$  value of the frequency magnitude relationship of earthquakes in map views in the Marmara Region from the corrected and declustered background seismicity catalogue based on  $M_D$  scale, covering the time period between 1981 and 1999. The computed spatial distribution shows strong spatial heterogeneity within the range  $0.84 \pm 0.03$ - $2.04 \pm 0.07$  in the region. Anomalously low  $b$  value areas more or less coincide with the spatial distribution of  $M \geq 5.5$  earthquakes and their known rupture extents. Then the local recurrence time,  $T_L$ , based on the frequency-magnitude distribution for a target event comparable to the 1999 İzmit earthquake ( $M_D$  6.7) is computed. Shortest  $T_L$  value of  $312(+186/-118)$  years is obtained in the Sea of Marmara, showing no much difference with the one derived from the historical observations. As the latest ruptures on the submarine segments have been reported to be during the 1754-1766 earthquake sequence, the mean recurrence time indicates an alarming seismic hazard for the eastern part of the Sea of Marmara in the near future.

## INTRODUCTION

The North Anatolian Fault Zone (NAFZ) is a 1500 km-long dextral transform fault that accommodates western extrusion of Anatolian plate resulting from the collision of the Arabian plate in the eastern Anatolia along with sinistral East Anatolian Fault Zone (Barka and Kadinsky-Cade, 1988; McClusky et al., 2000; Şengör et al., 2005) (Fig-1.). GPS velocity field indicated that virtually all of the Eurasian-Anatolian plate relative motion is accommodated along the NAFZ with a slip-rate of about 24 mm/year and virtually no internal deformation is observed within the central Anatolia (McClusky et al., 2000).

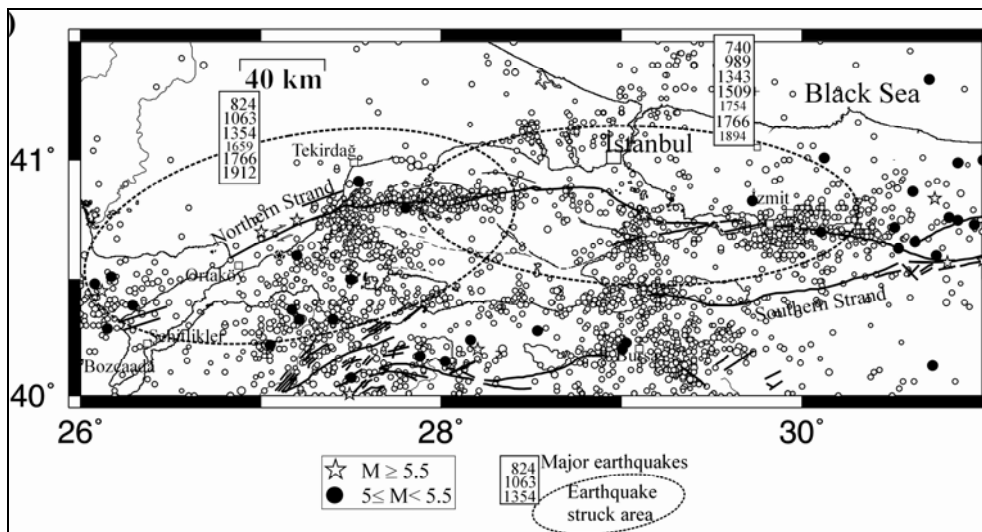
The NAFZ enters the Marmara region at its east section and bifurcates into two fault strands, the Northern and Southern strands (Barka and Cadinsky-Cade, 1988; Armijo et al., 2002) (Fig-2.). GPS and kinematics reconstruction studies have indicated that there is an asymmetric slip partitioning between the Northern and Southern strands with the Northern strand undertaking much larger motion than the Southern strand (Armijo et al., 2002; Flerit et al., 2003).

It has been determined that the Northern boundary fault accommodates 20 mm/year of the slip rate caused by the plate kinematics, that is about 24 mm/year, with no strike



**Figure-1.** The map showing major tectonic elements of Turkey (modified from Barka and Kadinsky-Cade 1988). Grey shaded area indicates the map area shown in Figure-2.

perpendicular motion, while the rest of the slip rate is accommodated along the Southern strand with prominent strike perpendicular motion or extension (Flerit et al., 2003). The both strands have produced tens of destructive large earthquakes over the past twenty centuries as revealed from the historical sources (Ambraseys 2002), indicating high seismic activity in the region.



**Figure-2.** Extent of North Anatolian fault and distribution of earthquake epicentres in the Marmara Region. The submarine faults are drawn as suggested by Armijo et al. (2002).  $5.0 \leq M_s \leq 5.5$  and  $M_s \geq 5.5$  earthquakes after 1900 are denoted with black solid circles and white stars, respectively. White filled circles represent epicentre map of the seismicity covering the time period of 1981-1999 and used in the study.

Following the rupture of 1999 İzmit earthquake the submarine section of the Northern strand defined as an apparent seismic gap that likely to produce large earthquake or earthquakes within the next decades (Parsons, 2004). Therefore the seismic hazard in the Marmara region has become a subject of general interest among the earth science community throughout the globe with special emphasise on the offshore section of the Northern strand (Le Pichon et al., 2001; Ambraseys, 2002; Flerit et al., 2003; Parsons, 2004; Armijo et al., 2005).

The Marmara region is a seismically active area with tens of destructive large earthquakes over the past twenty centuries as revealed from the historical sources (Ambraseys and Finkel, 1991 and 1995; Ambraseys and Jackson, 2000; Ambraseys, 2001; Ambraseys 2002) as well as paleoseismological studies (Klinger et al., 2003) (Fig. 2). Compiled  $M_S \geq 5.0$  seismicity in the instrumental period (after 1900) given in Table 1 with the epicentral distribution and the background two-decade-long  $M_D \geq 2.9$  microseismicity preceding 1999 İzmit earthquake (Fig-2.) verify high seismic activity in the region (Crampin and Üçer, 1975; Dewey, 1976; Ayhan et al., 1984; Ambraseys and Finkel, 1987; Kalafat et al., 2000; Öncel and Wyss, 2000; Özalaybey et al., 2002).

In the present study we mapped spatial variation of  $b$  values of the frequency-magnitude distribution (FMD) (Gutenberg and Richter, 1944; Wiemer and Wyss, 1997) in map view in the Marmara region and estimate FMD based earthquake recurrence times (Wiemer and Wyss, 1997) from the declustered background seismicity prior to the 1999 İzmit earthquake (Fig-2.) (Öncel and Wyss, 2000). Spatial distribution of  $b$  values and statistically estimated recurrence times are compared with the spatial distribution of moderate and large magnitude seismicity and historically observed recurrence times, respectively.

## DATA and METHOD

The data used in the study is corrected and declustered (Reasenberg, 1985), background seismicity catalogue (based on  $M_D$  scale, covering the time period between 1981 and 1999) that recorded by Kandilli Observatory and Earthquake Research Institute (KOERI), Turkish national network and other local network (Öncel and Wyss, 2000) (Fig-3.and 4.). Cumulative number of  $M_D \geq 2.9$  earthquakes in the catalogue shows roughly linear increase with time, which could be interpreted as that the catalogue homogeneous for the  $M_D \geq 2.9$  earthquakes.

The relationship between the size of an earthquake and its frequency of occurrence named as FMD (Gutenberg and Richter, 1944) and defined as:

$$\log_{10} N = a - bM \quad (1)$$

where  $N$  cumulative number of earthquakes with a magnitude exceeding a given magnitude  $M$ , and  $a$  and  $b$  are constants. The constant  $a$  is positively related to level of seismic activity. The  $b$  value has been shown to be inversely related to the shear stress in the crust (Wiemer and Wyss, 1997), is positively correlated with the increasing heterogeneity in the crust (Mogi, 1962) and shows strong heterogeneity in finer scales in the range 0.5-1.5. It has been shown that anomalously low  $b$  value along a fault zone corresponds to asperities that govern earthquake occurrence behaviour (Wiemer and Wyss, 1997).

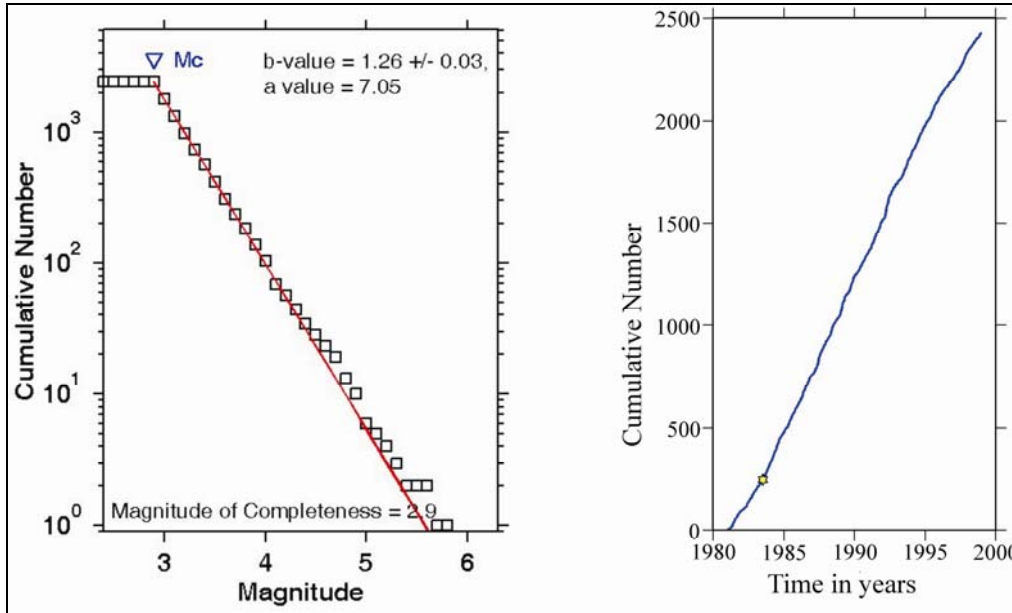
After determining the FMD relation from the seismicity catalogue of a certain region or fault zone recurrence time ( $T_r$ ) of an earthquake of targeted magnitude ( $M_{\text{targ}}$ ) can be practically estimated by

$$T_r = \frac{\Delta T}{10^{(a-bM_{\text{targ}})}} \quad (2)$$

where  $\Delta T$  is the recording period covered by the seismicity catalogue. When  $T_r$  is based on the finer scale distribution of FMD, its calculation in this way is called local recurrence time ( $T_L$ ) (Öncel and Wyss, 2000).

To calculate  $b$  value, we have used the maximum likelihood method (Aki, 1965):

$$b = \frac{\log_{10} e}{(M_{mean} - M_{min})} \quad (3)$$



**Figure-3.** Cumulative numbers of the earthquakes in the seismicity data covering the time period 1981-1999 are plotted against magnitude (left) and time (right). Note that the minimum magnitude of complete reporting is  $M_D$  2.9.

where  $M_{mean}$  is the average value of magnitude and  $M_{min}$  is the minimum magnitude of completeness in the seismicity catalogue to be analysed.

A software package called *ZMAP* is used for mapping  $b$  value of FMD and  $T_L$  value as a function of space (Wiemer and Wyss, 1997).

## RESULTS and DISCUSSION

We firstly used the cylindrical data volumes with radius of 13 km and height of 40 km that are centred at grid nodes separated by  $0.02^\circ$ . The minimum number of events for each data volume is set as 50. The computations are also implemented with different radiuses of 12 and 14 km. Virtually similar results are obtained with the spatial mapping area contracting with decreasing radius. The shortest and the largest  $b$  values are changing in the range 0.80-0.84 and 1.90-2.04, respectively, while shortest  $T_L$  values are varying in the range 212-349 years. We do not prefer any of these spatial distributions and base our discussion on absolute values and overall picture suggested by the calculations with different data volumes will be discussed. Anyway, the spatial distribution resulted from the analysis with a radius 13 km are shown in order to visualise the results (Fig-4.), with low and high  $b$  values are represented with blue and red colours, respectively. For this mapping  $b$  values are varying between  $0.84 \pm 0.03$  and  $2.04 \pm 0.07$  and the shortest  $T_L$  value 317 (+186/-118) years. Several anomalously low  $b$  value areas such as over the western half of the Sea of Marmara, around İzmit and Bursa and in the southern part of western Sea of Marmara is noticeable.

Since the anomalously low  $b$  values indicate high stress concentrations along a seismogenic zone (Wiemer and Wyss, 1997), one would expect correlation between the large earthquakes in the past and the patches of exceptionally small  $b$  values. To examine this we show epicentres of the  $M_S \geq 5.0$  earthquakes after 1900 compiled from the seismicity catalogues (Table-1.) and proposed and observed rupture zones of  $M_S \geq 6.0$  earthquakes after 1800 on our surface mapping in Fig-4. A striking correlation is naturally not accomplished because inadequate seismicity precludes  $b$  value computations in much of the mapping area.

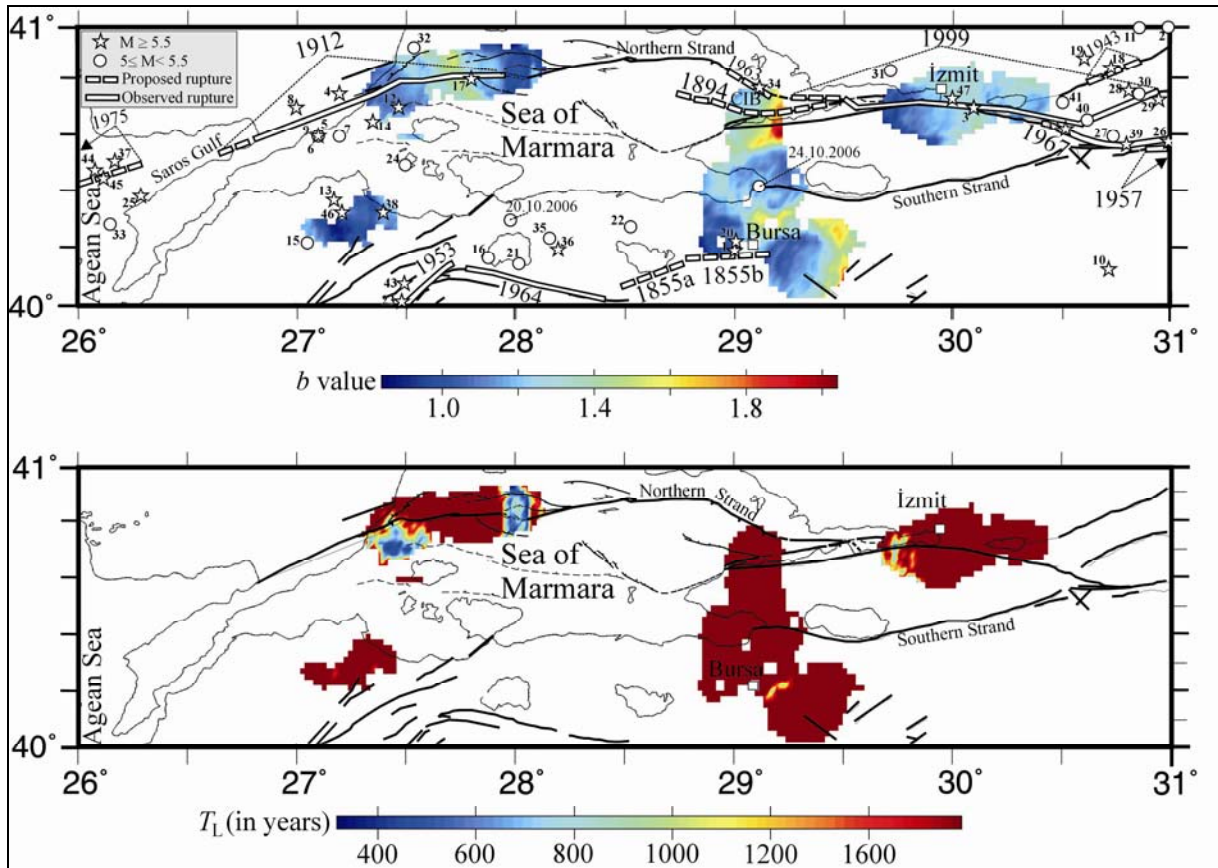
**Table-1.** The earthquakes with magnitude  $M_S \geq 5.0$  occurred along the NAFZ in the Marmara region, NW Turkey, since 1900. See Figure-2. for epicentre distribution.

No.	Date	Origin time	Latitude	Longitude	$M_S$	Ref.
1	15.04.1905	05.36.??	40.20	29.0	5.6	1
2	22.10.1905	03.42.??	41.00	31.0	5.2	1
3	21.08.1907	-	40.70	30.1	5.5	1
4	09.08.1912	01.29.??	40.75	27.20	7.4	2
5	10.08.1912	09.23.??	40.60	27.10	6.3	1
6	10.08.1912	18.30	40.60	27.10	5.3	1
7	11.08.1912	08.19.44	40.60	27.20	5.0	1
8	13.09.1912	04.27.??	40.70	27.00	6.9	2
9	10.04.1917	19.40.18	40.60	27.10	5.3	1
10	16.12.1926	17.54.05	40.13	30.72	5.7	1
11	24.01.1928	07.36.11	40.99	30.86	5.3	1
12	04.01.1935	14.41.30	40.70	27.47	6.4	3
13	04.01.1935	15.19.18	40.37	27.17	5.6	3
14	04.01.1935	16.20.04	40.65	27.35	6.3	3
15	22.10.1935	07.29.42	40.22	27.05	5.2	1
16	02.07.1938	12.26.45	40.17	27.88	5.0	1
17	16.06.1942	05.42.34	40.80	27.80	5.6	1
18	20.06.1943	15.32.54	40.84	30.73	6.6	1
19	20.06.1943	16.47.59	40.87	30.61	5.5	1
20	13.11.1948	04.44.50	40.23	29.02	5.6	1
21	15.09.1951	22.12.52	40.15	28.02	5.0	1
22	03.06.1953	16.05.31	40.00	28.43	5.3	1
23	18.03.1953	19.06.13	40.01	27.49	7.2	4
24	23.03.1954	12.58.46	40.50	27.50	5.0	1
25	06.01.1956	12.15.44	40.39	26.29	5.5	1
26	26.05.1957	06.33.35	40.58	31.00	7.0	4
27	26.05.1957	08.54.51	40.60	30.74	5.4	1
28	26.05.1957	09.36.38	40.76	30.81	5.9	1
29	27.05.1957	11.01.34	40.73	30.95	5.8	1
30	01.06.1957	05.26.59	40.75	30.86	5.0	1
31	26.12.1957	15.01.44	40.83	29.72	5.2	1
32	26.07.1959	17.07.06	40.91	27.54	5.4	1
33	29.03.1963	03.09.17	40.29	26.15	5.1	1
34	18.09.1963	16.58.14	40.77	29.12	6.3	1
35	06.10.1964	14.29.57	40.24	28.16	5.1	1
36	06.10.1964	14.31.23	40.20	28.20	6.8	4
37	23.08.1965	14.08.58	40.51	26.17	5.6	1
38	21.08.1966	01.30.43	40.33	27.40	5.5	1
39	22.07.1967	16.56.58	40.57	30.80	6.9	4
40	22.07.1967	17.48.06	40.66	30.62	5.1	1
41	22.07.1967	18.09.55	40.72	30.51	5.0	1
42	30.07.1967	01.31.01	40.63	30.53	5.6	4
43	03.03.1969	00.59.10	40.08	27.50	5.7	1
44	25.03.1975	02.52.52	40.48	26.08	5.8	1
45	27.03.1975	05.15.07	40.45	26.12	6.7	1
46	05.07.1983	20.49.33	40.33	27.21	5.8	5
47	17.08.1999	00.01.38	29.97	40.73	7.8	6,7

<sup>1</sup> Ayhan et al. (1984); <sup>2</sup> Ambraseys and Finkel (1987); <sup>3</sup> Crampin and Uçer (1975); <sup>4</sup> Dewey (1976); <sup>5</sup> Kalafat et al. (2000); <sup>6</sup> Özalaybey et al., 2002; <sup>7</sup> United States Geological Survey.

However, several evidences for good correlations can be mentioned. The rupture zone of the 1999 İzmit earthquake with major seismic moment release (Pinar et al., 2001) and the submarine section of the 1912 break crosses low  $b$  value patches. The June 21, 1907  $M_S$  5.5 (number 3 in Fig-4.) earthquake occurred inside low  $b$  value patches along

the rupture zone of the 1999 İzmit mainshock. Interestingly, two largest shocks of the 1935 earthquake sequence measured  $M_S$  6.4 and  $M_S$  6.3 (numbers 12 and 14 in Fig-4.) took place inside and rim of the anomalously low  $b$  area in the western Sea of Marmara, respectively. Mapped low  $b$  area in the western Sea of Marmara also host June 16, 1942 event with  $M_S$  5.6 (number 17 in Fig-4.).



**Figure 4.** Map view of spatial distribution of  $b$  values in the Marmara region with overlay of  $5 \leq M_S < 6.0$  earthquake epicenters after 1900 (white filled circles and stars) and the observed and proposed rupture extends (continuous and broken thick white lines, respectively) of  $M_S \geq 6.0$  earthquakes after 1800 (top). Numbers near the earthquake epicentres denote the earthquake numbers in Table-1. Map of local recurrence times ( $T_L$ ) for a target event of  $M_D$  6.7, which is measure of the 1999 İzmit earthquake ( $M_W$  7.4) in  $M_D$  scale (bottom). Calculations are implemented using cylindrical data volumes with radius of 13 km and height of 40 km that are centred at grid nodes separated by 0.020 spacing. The minimum number of events is required for each estimate is 50. CIB: Çınarcık basin.

The largest shock in the seismicity catalogue used for spatial mapping of  $b$  value in the study, the July 5, 1987  $M_S$  5.8 (number 46 in Fig-4.) earthquake with several other  $5 \leq M_S < 6.0$  earthquakes took place within or edge of the one of the low  $b$  value patch in the Asiatic side of western Sea of Marmara. The large historic 1855b earthquake rupture follows southern rim of a low  $b$  value patch west of Bursa. This patch also host two  $M_S \geq 5.5$  earthquakes (numbers 1 and 20 in Fig-4.).

Armijo et al. (2005) pointed out that the 1999 İzmit earthquake rupture continues to the eastern entrance of Çınarcık basin (CIB) as seen in Fig-4. Our surface mapping indicates

anomalously high  $b$  value at this location. As the  $b$  value inversely related to the stress in the seismogenic crust, the low stress area mapped from the seismicity two decades before the 1999 İzmit earthquake might have stopped the rupture of this earthquake at the entrance of CIB. Note that the  $b$  values get lower over the eastern halves of the fault segments bounding CIB as getting west from this anomalously high  $b$  value patch. Though sparse seismicity do not allow estimation over the western halves of the bounding faults, the possible ruptures of the both 1894 and 1963 earthquakes (see Armijo et al. 2005) are partly covered with this low  $b$  value area. The epicentre of the 1963 earthquake (number 34 in Fig-4.) was also located in low  $b$  value area on the fault bounding northern edge of CIB.

In spite of the fact that scarcity of the seismicity data do not allows  $b$  value calculations in much of the mapping area in concern, Fig-4. shows that almost half of the  $M_S \geq 5.5$  earthquake epicentres after 1900, including 1999 İzmit earthquake epicentre, were located within or edge of the exceptionally low  $b$  value areas. Note that one of the moderate shocks (October 24, 2006 earthquake) that struck south of the Marmara region in 2006 also located in the low  $b$  anomaly north of Bursa.

Using the estimated spatial variability of FMD, recurrence times for a magnitude  $M_D$  6.7 event are calculated according to Eq. (2) for each grid node. The selection of target event magnitude as  $M_D$  6.7 is due to the fact that the latest destructive shock in the study area, the 1999 İzmit earthquake, was measured as magnitude 6.7 event based on the  $M_D$  magnitude scale by KOERI. Note that the shortest  $T_L$  anomaly down to 312 years is mapped along the Northern strand over the western Sea of Marmara. This value is more or less same as the one derived from historical observation indicating that the large historical shocks affecting eastern Sea of Marmara are 740, 989, 1509 and 1766 earthquakes and western Sea of Marmara 863, 1063, 1354, 1659, 1766 and 1912 (Fig-2.). It could be asserted that the mean earthquake return period along the offshore section of Northern strand is about 250-300 years. As the latest ruptures on the submarine segments have been reported to be during the 1754-1766 earthquake sequence, the mean recurrence time indicates an alarming seismic hazard for the eastern part of the Sea of Marmara in the near future.

Therefore, the spatial mapping of  $b$  values and related  $T_L$  maps can be used as a tool for discussing future seismic hazard and interpreting more likely regions for future earthquakes. However, much testing through new studies is still needed.

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