ESTIMATION OF THE OCEAN TIDE EFFECT ON THE ALTIMETRIC MEASUREMENTS

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ABSTRACT *The oceanic tide is a periodic phenomenon of rise and fallen of the sea level. It is governed by the gravitational action of the solar system bodies essentially the moon and the sun, it translates by a transport of water masses. The evaluation of the oceanic tide is calculated using two global models GOT00.2 and FES95.2; the comparison of the results obtained with the ocean tide value transmitted in the message of Topex/Poseidon satellite allowed us the validation of the methodological approach developed. The proximity of the results obtained is sufficient for the most altimetric applications as the determination of the mean sea level which was calculated on the western Mediterranean Sea during a 72 cycle period.*

INTRODUCTION

Earth incur phenomena arrived of the outside (the solar system), among these phenomena the tides effect that are result of the effect of the luni-solar attraction strength on the earth. There are a many types of tide: the ocean tide, the solid tides and the polar tide.

Our work is based on the evaluation of the ocean tide effect on the altimetric measurement, used two main models GOT00.2 and FES95.2; these two models are the empiric models, based essentially on altimetric measurements After, we correct the altimetric measurement of the ocean tide effect, the comparison of our results with those given in the message of Topex-Poseidon satellite permit us to validate our methodology. The results are used to determinate the Mediterranean mean surface.

OCEAN TIDE

Generating Ocean Tide Strength

The survey of tide comes back to interest at the liquid particles movements of oceans and seas of the terrestrial globe in a geocentric coordinates reference. Indeed, these particles are submitted to the gravitational force of the terrestrial attraction and the force of star attraction at a time in revolution around the Earth, called disruptive stars. We distinguish principally the effect of attraction of the sun (owed to its mass) and of the moon (because its proximity).

This force drag variations of the level of ocean and seas of the globe that vary in the time and that form waves of propagation of the tide; these force are called: generating forces.

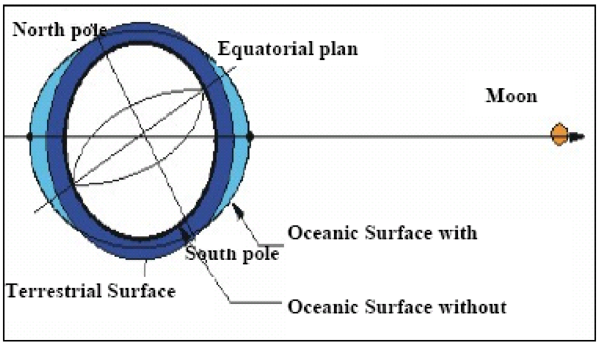

Figure-1. Influence of a star on the surface of oceans and seas

Laws of the Newtonian mechanic permit us to calculate the force of attraction between the earth *T* and any star *K* by (Newton, 1687):

$$\overrightarrow{F_{T/K}} = -G\, m_T m_A \frac{\overrightarrow{D_{T/K}}}{D_{T/K^3}} \qquad (1)$$

With:

•*G*: Constant gravitational ;

•$D_{T/K}$: Direction vector: the line passing by the earth *T* and the star *K*.

Let's recall that the expression of the gravitational constant is according to the field of attraction terrestrial g that is given by:

$$G = g.\frac{a^2}{m_T} = 6{,}667.10^{-24} \ \mathrm{m}^2 s^{-2} kg^{-1} \qquad (2)$$

With:

•*a*: Earth radius ($a \approx 6400$km);

•m_T: The Earth mass ;

•m_K: The star mass.

Direction and module

By a simple geometric construction, we are going to determine the direction and the module of the generating force of tide for a disruptive star (Newton, 1687).

Let the Earth *T*, a star *K*, *A1* and *A3* two points of (*OC*) and *A2* and *A4* two points of (*PC*) constructed as :

- *C* is the centre of the star *K*.
- the arc of circle PA_1C either of centre *C* and radius *PC* either r_k ;
- the right (*A1A2*) is parallel to (*PT*) ;
- the arc of circle *A2A3* either of centre *C* and radius *A2C;*
- the right (A_3A_4) is parallel to (A_1A_2);
- the arc of circle A_4CP' either of centre C and radius A_4C.

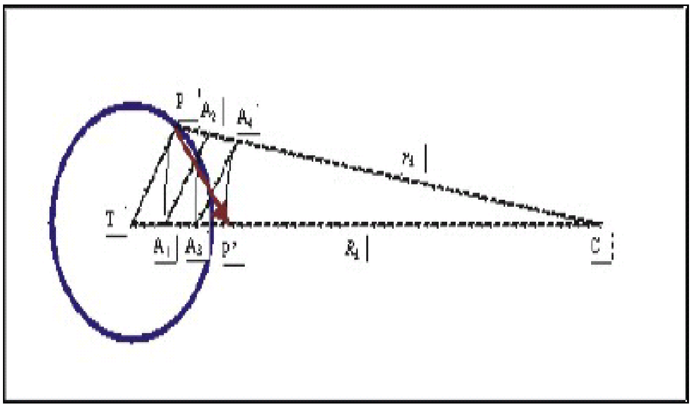

Figure-2. Geometric construction of the generating force of tide

Therefore we have by construction:

$CP' = CA_4$, $CA_3 = CA_2$, $CA_1 = CP = r_k$ (3)

And by the Thalès theorem we are:

$$\frac{CA_2}{CP} = \frac{CA_1}{CO} \Rightarrow CA_2 = CA_1 \frac{r_k}{R_k} = \frac{r_k^{\,2}}{R_k}; \frac{CA_4}{CP} = \frac{CA_3}{CO} \Rightarrow CP^{'} = CA_4 = CA_3 \frac{r_k}{R_k} = \frac{r_k^{\,3}}{R_k^{\,2}} \quad (4)$$

Of where:

$$\overrightarrow{PP^{'}} = \overrightarrow{PC} + \overrightarrow{CP'} = r_k \overrightarrow{V_k} - \frac{r_k^{\,3}}{R_k^{\,2}} \overrightarrow{u_k} = r_k^{\,3} \left(\frac{\overrightarrow{V_k}}{r_k^{\,2}} - \frac{\overrightarrow{u_k}}{R_k^{\,2}} \right) \quad (5)$$

Then in any point P of the ocean surface, the generating force of tide $\overrightarrow{F}_{\mathrm{GM}}$ of a star k, with a mass m_K and a centre C and a R_K distance, is given by: (Newton, 1687)

$$\overrightarrow{F_{GM}} = \frac{G\,\mathrm{m_k}}{R_k^{\,3}} \overrightarrow{PP^{'}} \quad (6)$$

This tracing is simplified thanks to the rule of Proctor that, supposing the star *K* is to the infinite, train that the arcs of construction circle are some parallel rights and therefore $TP'=3TA_1$.

We are going to simplify Eq. (6) while expressing $\overrightarrow{\mathrm{PP'}}$ according to *a (*the Earth radius) and θ (the zenithale distance of the star *K)*. By the rule of Proctor, we have:

$$OP^{'} = 3\,OA_1 = 3\,a\cos(\theta) \quad (7)$$

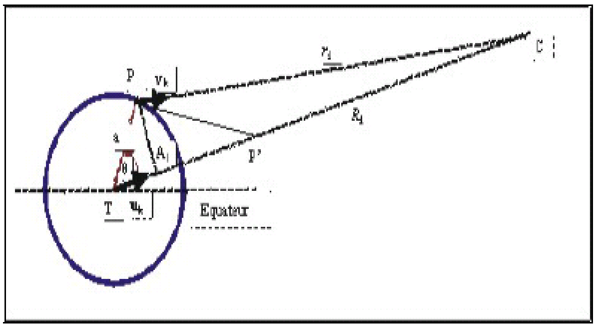

Figure-3. Angular expression of the generating force of tide

By the theorem of Pythagore in a rectangle triangle, we have:

$$PP'^2 = PA_1^{\,2} + A_1P'^2 \quad (8)$$

That gives:

PP'^2=(a sin(θ))^2+(3 a cos(θ)-a cos(θ))^2=a^2 (cos^2 (θ)+1) (9)

From where the expression of the module of the generating force according to θ the zenithale distance of the star:

$$\left\|F_{GM}\right\| = G\frac{m_k}{R_k^{\,3}}a\sqrt{3\cos^2(\theta)+1} \quad (10)$$

Thanks to this equation and to the method of tracing defined in the previous paragraph, we can define the surface that would take a water layer regaining the Earth by the influence of a unique star, (Fig.1) watch well this influence. This surface is symmetrical with regard to the poles axis on the one hand and with regard to the axis of 'centre of the Earth - centre of the star' on the other hand, then this surface is an ellipsoid of revolution (Newton, 1687).

Generating Potential

The generating force of tide is a derivative force of a potential called the tide-generating potential, which is given under the form: (Newton, 1687)

$$\Pi_A = \frac{4}{3}c_L\sum_{n=2}^{\infty}\left[\left(\frac{R_{0L}}{R_L}\right)^{n+1}\left(\frac{a}{R_{0L}}\right)^{n-2}P_n(\cos(\theta_L))\right] + \frac{4}{3}c_S\sum_{n=2}^{\infty}\left[\left(\frac{R_{0S}}{R_S}\right)^{n+1}\left(\frac{a}{R_{0S}}\right)^{n-2}P_n(\cos(\theta_S))\right] \quad (11)$$

Where:

$P_n(x)$ is the Legendre polynomial; C_L and C_S are the geodesic coefficients of Doodson for the Moon and the Sun defined by:

$c_L = \frac{3}{4}G\,\mathrm{m_L}\frac{a^2}{R_{0L}^{\,3}}$ and $c_S = \frac{3}{4}G\,\mathrm{m_S}\frac{a^2}{R_{0S}^{\,3}}$

Where:

•R_{0L}: the mean distance between Earth and Moon;

•R_{0S} : the mean distance between Earth and Sun;

The generating potential can be written in hourly coordinates like following (Lefeuvre, 2000):

$$\Pi_L = c_L\left(\frac{R_{0L}}{R_L}\right)^3\begin{pmatrix}\cos^2(\varphi)\ \cos^2(\theta)\ \cos(2Ah)\ + \\ \sin(2\varphi)\ \sin(2\theta)\ \cos(Ah)\quad + \\ \frac{1}{3}(3\cos^2(\varphi)\text{-}1)\,(3\cos^2(\delta)-1)\end{pmatrix} \quad (12)$$

The three terms of the sum in Eq. (12) have a different dependence according to *Ah*

• The term cos^2 (φ) cos2 (θ) cos(2Ah) , depending of *2Ah*, is minimum and maximum lasting a revolution of the star around the Earth on two East eridian opposed of 90° (and their complementary respective to the West) by revolution of the star around the Earth, and therefore its periodicity is semi-diurnal and its sign depends only the sign of cos(*2Ah*). Therefore, on the same meridian its sign remains constant. We speak of sectorial spatial distribution.

• The termsin(2φ) sin(2θ) cos(Ah) , depending of *Ah,* is once minimum and maximum on East meridian (and his complementary on West) for one revolution, its periodicity is diurnal and its sign depends the position of the star. We speak of tesseral spatial distribution.

• The term $\frac{1}{3}(3\cos^2(\varphi)\text{-}1)\,(3\cos^2(\delta)-1)$ doesn't depend of *Ah*, we say that it is a long period term. We speak of zonal spatial distribution.

Development of the Potential

Thanks to the decomposition of the generating potential in Legendre polynomials, we expressed the potential in an infinite series by Eq. (12). Besides, as we saw him higher, it is possible to express movements of the disruptive stars responsible for the tide generation (as the occurrence for the terrestrial system, the Moon and the Sun) of very precise and linear manner. It is why it is possible to develop this generating potential in set of functions pseudo harmonic (development of Darwin) or harmonic (development of Doodson) (3). We intend to give developments of this generating potential under the form of sinusoidal functions of the time or not, all dependent of the position observation coordinates. This potential goes contained terms classified according to three types of tides: long period terms, diurnal terms and semi-diurnal terms.

The representation of these waves (amplitude function of the considered frequency of the wave) constitutes the tide spectre determined from ocean level elevations in the time in a given point. The spectral analysis permits to differentiate the different composantes and the harmonic analysis (mathematical count based on signal theory) provides the amplitude and the phase of each of these composantes (Lefeuvre, 2000).

Darwin Development

Darwin presented the first development of the tide-generating potential in sinusoidal functions of the time. It is almost-harmonic because it contains the pseudo constant that vary very slowly in the time. (Darwin, 1883)

This development contains 32 lunar terms and 59 solar terms, every wave is characterized by a composed symbolic appellation of a letter to indicate the group to which it belongs (M, S, OH, N, K, Q,) and by an indication to specify its periodicity (Darwin, 1883):

• a : for annual ;

• m : for monthly ;

• f : for fortnightly;

• l : for diurnal ;

• 2 : for half-diurnal.

• 3 : for third-diurnal…

Doodson Development

In 1921, Doodson presented a development of the generating potential more complete. While leaning on the lunar theory of Brown, he expressed coordinates of the Moon withregard to the ecliptic. These developments, contrarily to those of Darwin, are merely harmonics. They drive to about 400 composanteses of the potential. Doodson used the 5 fundamental angles as well as the middle lunar time to position movements of the Moon and the Sun in the copernicien terrestrial reference (Doodson, 1921).

The potential Π_2 (developped to the order 2 of the set in Legendre polynomials) is expressed then under the form (Doodson, 1921):

$\Pi_A=\Pi_{20}+\Pi_{21}+\Pi_{22}$ (13)

With:

• For the long periods waves :

$$\begin{cases}\Pi_{20}=\lambda_0\left[c_L\left(\frac{R_{0L}}{R_L}\right)^3\cos^2(\delta_L)\cos(2AH_L)+c_S\left(\frac{R_{0S}}{R_S}\right)^3\cos^2(\delta_S)\cos(2AH_S)\right.\\ \lambda_0=\frac{1-3\sin^2(\varphi)}{2}\end{cases} \quad (14)$$

• For the diurnals waves:

$$\begin{cases}\Pi_{21}=\lambda_1\left[c_L\left(\frac{R_{0L}}{R_L}\right)^3\sin(\delta_L)\cos(AH_L)+c_S\left(\frac{R_{0S}}{R_S}\right)^3\sin(\delta_S)\cos(AH_S)\right.\\ \lambda_1=\sin(\varphi)\end{cases} \quad (15)$$

• For the half-diurnals waves:

$$\begin{cases}\Pi_{22}=\lambda_2\left[c_L\left(\frac{R_{0L}}{R_L}\right)^3\left(\frac{2}{3}-2\sin^2(\delta_L)\right)+c_S\left(\frac{R_{0S}}{R_S}\right)^3\left(\frac{2}{3}-2\sin^2(\delta_S)\right)\right.\\ \lambda_2=\cos^2(\varphi)\end{cases} \quad (16)$$

For where:

• λ_i is factors of latitude: who determines the type of the tide. They depend on the latitude to which the potential was exercises;

• δ_L and δ_S are the respective declensions of Moon and Sun;

• AH_L and AH_S are the respective hourly angles.

Estimation of the Ocean Tide

We can summarize that the tide can be considered as being the sum of strictly periodic elementary tides called harmonic Coordinates. The tide curve of a wave is a sinusoid of which the amplitude and the phase only depend on the place of observation. So the height of the tide to a t instant can be express theoretically by the following Formula (Lefeuvre, 2000):

$$H_{OT}(\varphi,\lambda,t)=\sum_i Z_i(\varphi,\lambda)\cos(\omega_i.t-\Psi(\varphi,\lambda)) \quad (17)$$

• Z_i : is the amplitude of the wave I;

• ψ_i : is the phase expressed to the instant of passage of the disruptive body (moon or sun) to the meridian of Greenwich;

• ω_i : is the frequency of the composante k : it is given by the development of Doodson.

For any place, the two quantities *Zi* and ψ *i* that depend the position only (λ ,), are extracted from a global models. Among the first models that appeared, we find the

FES95.2 and SCR3.0; these models are files of a matrix form (under form of a grid of the longitude and the latitude). The amplitude and the phase of the wave can be calculated thus by a bilinear interpolation.

The model FES95

This model is merely a streamlined model; it is suited to adjust measures of tides globally. it has been developed while leaning on the theory of finished elements in order to have a good precision close to quote them, the FES95.2 version is a grid of 0.5°×0.5°, it contains the Topex/Poseidon altimetric satellite data, that has been used to adjust the behaviour of strong length waves (User Handbook,2003).

The models CSR3.0 and CSR4.0

Are adjustments of the fortes lengths waves for FES95 while using the supplementary data of Topex/Poseidon, these models are given on a grid of 0.5°×0.5°.But unfortunately, they have cells that fall on soil and that have been removed while using the grid of the model GOT 00.2 like a mask (User Handbook, 2003).

PROCESSING ANALYSIS

Data Processing

To validate the estimation of tide effects, altimetric data of the pass 187 cycle 309 of the Topex/Poseidon satellite is used and the gotten results are presented below:

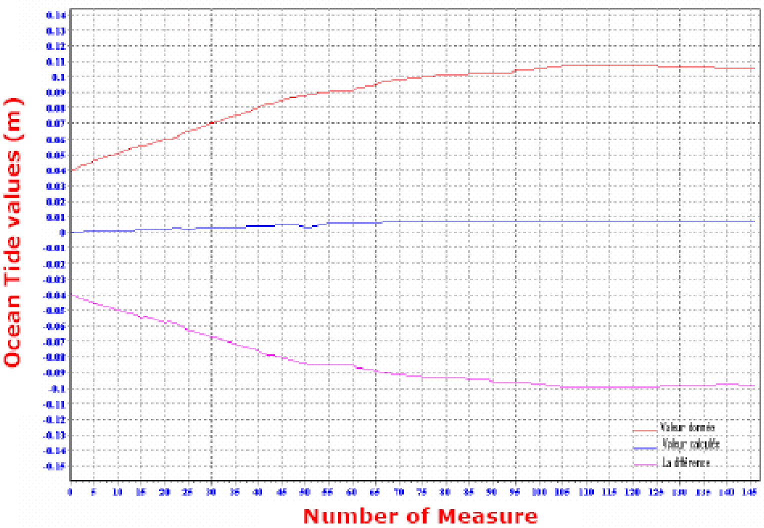

Figure-4. Difference between oceanic tide stocked by the message and calculated oceanic tide by the FES95.2 model (p187c309)

Table-1. Means and standard deviation

	Mean (m)	Standard deviation
Ocean Tide Provided	0.0892	0.0199
Ocean Tide Calculated	0.0054	0.0020

Analysis of results

The graph that represents the calculated oceanic tide effect for the Topex/Poseidon satellite is baffled to the one that represents the same effect provided by the message, notably when we compare the means and Standard deviation that present important differences (few centimetres). This difference is owed to the chosen model mainly, because one used the model FES95.2 on the other hand well stocked oceanic tide values are gotten while using the model FES99 and also to the problem of values by defect that influential at the time of bilinear interpolation.

But we can say that this output is sufficiently good to can use them to end to determine the middle level of the Mediterranean western.

THE MEAN SEA LEVEL DETERMINATION

The determination of the altimetric geoïd in the western Mediterranean is done from the Topex/Poseidon data corrected of the different effects used in the precedent model for the correction of the sea state bias effect.

The used data (*GDR-M* pass files) are those well stocked by Aviso on *CD*: *« Jason-1 and TOPEX/POSEIDON GDR products »* from April 4th, 2002 to the March 28th, 2004, corresponds to 72 cycles.

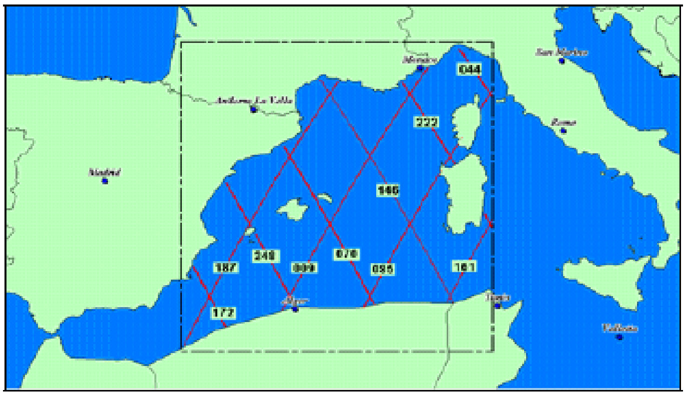

Figure.5. Traces of the Topex/Poseidon satellite covering the processing zone

Deviation Correction on the Crossover Points

For the crossover points coincides two different measures of the sea level. The deviation between these two measures must be corrected and must be distributed on the whole of the two profile measures.

The used method for the distribution of this deviation on altimetric profiles is the polynomial interpolation method, whose principle is as following (Rami, 2006):

Let$_N$ a crossover point of one same altimetric profile $y_1=f(x_1)$, $y_2=f(x_2)$,........,$y_N=f(x_N)$ where the y_i represent corrections to bring to the crossover points and x_i the longitudes of the crossover points. The orbital correction for a point of the longitude x profile will be expressed then by the Lagrange classic formula as follows:

$$y=\frac{(x-x_2)(x-x_3)...(x-x_N)}{(x_1-x_2)(x_1-x_3)...(x_1-x_N)}y_1+\frac{(x-x_1)(x-x_3)...(x-x_N)}{(x_2-x_1)(x_2-x_3)...(x_2-x_N)}y_2+...+\frac{(x-x_1)(x-x_2)...(x-x_{N-1})}{(x_N-x_1)(x_N-x_2)...(x_N-x_{N-1})}y_N$$

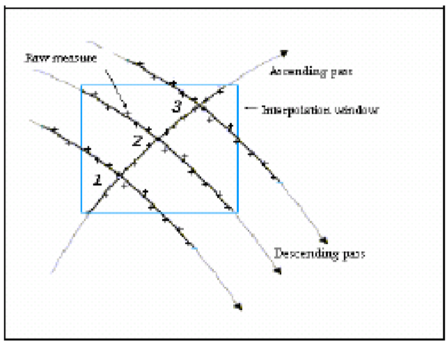

Figure-6. Crossover point between ascending pass and descending pass

Model of Processing the Altimetric Geoïd

The formulation of the processing model of the altimetric geoid height (*N*) is given as follows (Rami et al, 2006):

$$N = Hp\_Sat - (H\_Alt + \Sigma) \quad (18)$$

Where:

Hp_Sat: Altitude of satellite above the reference ellipsoid;

H_Alt: Altimeter range;

Σ: Whole corrections to be added to the altimeter range, given by: (Rami, 2006)

$$\sum = CG\_Rang\_Corr + dry\_Corr + Wet\_Corr + Iono\_Corr\_K1 - SSB + INV\_Bar + H\_Eot\_Fes + H\_Set + H\_Pol \quad (19)$$

Where:

CG_Range_Corr: Correction to the altimeter tracker range for gravity centre movement.

Dry_Corr: Dry tropospheric correction;

Wet_Corr: Wet tropospheric correction;

$Iono\_Co$: Ionospheric correction;

SSB: Sea state bias;

INV_Bar: Inverse barometer correction for altimeter measurement;

H_Eot_FES: Height of the elastic ocean tide at the measurement point computed from

FES 95.2 model;

H_Set: Height of the solid earth tide at the measurement point;

H_Po : Geocentric pole tide height at the measurement point.

Results

The nappage of the mean profiles corrected of the orbit error by a regular grid (0.25°x0.25°) (in longitude and in latitude) permits to have a mean sea surface.

The used method is the linear interpolation (triangulation of DELAUNAY); who was used to exclude the regions that have not been observed by altimetry (Kahlouche et al, 2003).

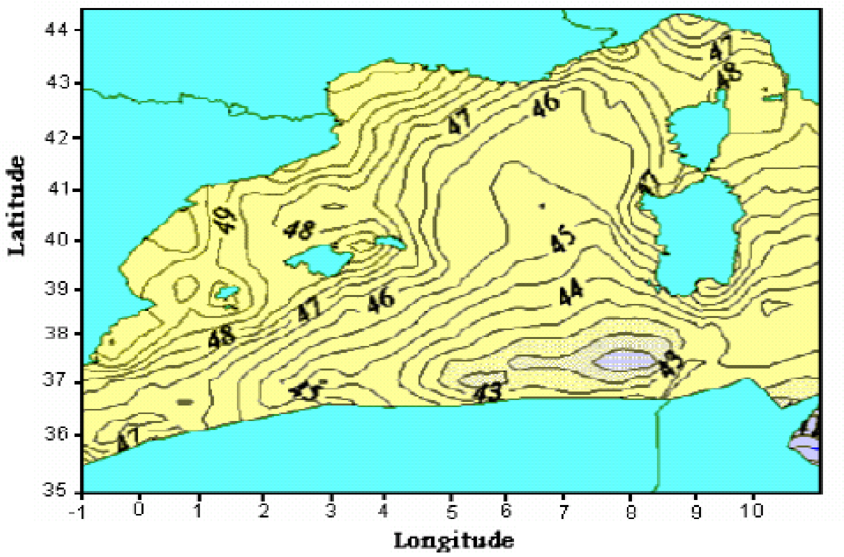

Figure-7. Altimetric geoid height determined from Topex/Poseidon

CONCLUSION

The estimation of ocean tide effect as using the model FES95.2 permitted us to say that this model is not very effective at 100% and require to be improving by implication of other altimetric measurements.

The use of other models for example FES99 or FES200 will give us forcing a very small gap between the calculated tides and those stocked in the message and therefore we will have a better determination of the mean sea surface.

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