CANCELING STATIONARY SINUSOIDAL NOISE FROM MAGNETOTELLURIC’S RAW TIME SERIES


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ABSTRACT Sinusoidal noise often contaminates Magnetotelluric data. When this noise is large compared to MT signals, it adversely affects on Time series processing and subsequent interpretation. We develop a digital least squares filtering algorithm for canceling stationary sinusoidal noise in MT data. The method effectively cancels sinusoidal noise when the noise is stationary; this procedure differs from the usual notch-filtering techniques because the sinusoidal noise is canceled without notching the signal spectrum. Since the method requires that the power line frequency be accurately known, the algorithm can automatically search the Time series spectrum to find the exact sinusoidal frequency value needed for filter design. The algorithm is highly automated and requires no input parameters when the interference comes from power lines, generators or Railways.

INTRODUCTION

Stationary sinusoidal noise often contaminates MT data. In data acquisition power-line interference corrupts the MT recording with 50 Hz sinusoidal noise and higher harmonics. Rotating machinery can also create this type of interference. There are several methods of attenuating this type of interference. In the field, analog notch filters can be applied. Unfortunately this method also attenuates signals at the notch frequency, as well as frequencies around the notch point. Alternatively, automatic balancing circuits as described in Smither and Pater (1982) and Ensing (1983) can also be used to automatically adjust two potentiometers on each channel in the MT recording system to minimize power line interference. With these methods, the sinusoidal interference can be attenuated without removing signal energy of the same frequency. Modern practice is to record the interference and then remove it in processing. In processing, there are several ways of removing sinusoidal interference. Spiking deconvolution can be applied to flatten the spectrum, thereby reducing the amplitude of the power line interference. However, signal distortion results when using this method, so it is not recommended. A digital notch filter can be applied but, as with the analog notch filter, signal frequencies around the power line frequency are also attenuated. Alternatively a noise-canceling method can be used to remove the sinusoidal interference without distorting the signal at and around the power line frequency. In this paper, we present a different method of removing stationary sinusoidal interference. In our method, we assume the noise comprises a stationary sinusoid of a single frequency. We generate an analytic sinusoidal time series at the frequency of the interfering noise. A short Wiener least-squares filter is then used to match the amplitude and the phase of the sinusoid with the stationary sinusoidal noise in the data. The sinusoid is then subtracted from the corrupted data leaving the signal. Multiple sinusoidal power line frequencies can be canceled by repeated filter design and application. Our method is different from methods used in the past. The least-squares filter is optimally designed to provide the least amount of prediction error. The lengths of the autocorrelation and cross correlation functions are also optimally designed for the specific power line frequency to be attenuated. Taper on and taper off end effects due to filter application are automatically eliminated without distorting the signal at the beginning and end of each time series. The algorithm requires that the frequency of the
interfering noise be known exactly. So a method is used to automatically search the frequency spectrum of the data and calculate the exact frequency of the noise. Although the algorithm requires that the interfering sinusoidal noise be stationary over the recording length, we do not feel that this is a severe limitation. The Wiener least-squares filter used in our algorithm will not do this if it is designed over a portion of the time series where the noise is large. Unlike notch filtering, this noise canceling method does not notch the signal spectrum and thus can provide a noise-free undistorted signal spectrum. However, it requires more computer time than simple notch filtering and therefore is more costly. In this paper, we first present the theory and implementation of the sinusoidal noise canceller. And finally, we quantify the algorithm’s performance through use of field data examples.

THEORY

In our method a reference sinusoidal noise and a corrupted Time series are used to design an optimum Wiener filter. The Wiener filter is then used to adjust the amplitude and phase of the reference sinusoidal noise with the sinusoidal noise in the corrupted Time series. The reference sinusoidal noise is then subtracted from corrupted Time series removing the sinusoidal noise. The algorithm hinges around the Wiener filter design. The use of Wiener optimum filters has been the subject of many papers in the field of digital seismic signal processing. Norbert Wiener (1949, first published in 1942) developed a mathematical theory providing the fundamental principle for seismic deconvolution. Digital least-squares filter design involves solving the so-called normal set of linear simultaneous equations. Levinson (1947) developed a fast recursive algorithm to solve the normal equations for a Wiener optimum filter. This is referred to as the Wiener-Levinson algorithm. In our application, we assume the Time series $x_i$ comprises a signal $T_i$ and sinusoidal noise $n_i$: 

$$x_i = T_i + n_i, \text{for } i = 0 \ldots T$$

We assume that we know the frequency $f$ of the contaminating noise and define a sinusoidal Time series of length $T$ samples:

$$S_i = \sin (2\pi A_i f), \text{for } i = 0 \ldots T.$$  

(2)

Where $A_i$ = sampling interval. The phase and amplitude of $S_i$ does not match that of $n_i$ so it cannot be subtracted out at this step. We need a filter $H$, that when convolved with $S_i$ will adjust the amplitude and phase of $S_i$ to match the contaminating noise $n_i$.

$$S_i = \sum_{n=0}^{N-1} H_n S_{i-n}$$

(3)

Using only part of the data $x_i$ where the sinusoidal noise is larger than any other signal on the Time series, we define the error as follow:

$$E_i = x_i - s_i$$

(4)

And a cost function
\[ Q = \sum (E_i)^2 \] 
(5a)

Or
\[ Q = \sum x_i - s_i \] 
(5b)

\[ Q = \sum x_i^2 - 2 \sum x_i \sum H_n s_{i-n} + \sum (\sum H_n s_{i-n})^2 \] 
(6)

Taking the partial derivative of \( Q \) with respect to the coefficients of the filter \( H \) and setting them (i.e., the partial derivatives) equal to zero, we obtain:

\[ \sum x_i s_{i-m} = \sum H_n \sum s_i s_{i+n-m} \] 
(7)

The first summation in the above equation is the cross correlation, of the corrupted time series \( x_i \) with the sinusoidal reference noise \( S_i \). The second summation is the autocorrelation \( R_{m-n} \) of the sinusoidal reference noise convolved with the filter. This can be put in the following matrix form

\[ C_m = \sum H_n R_{n-m} \] 
(8a)

\[ \begin{pmatrix} R_0 & R_1 & \ldots & R_{N-1} \\ R_1 & R_2 & \ldots & R_{N-2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N-1} & \ldots & \ldots & R_0 \end{pmatrix} \begin{pmatrix} H_0 \\ H_1 \\ \vdots \\ H_{N-1} \end{pmatrix} = \begin{pmatrix} C_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{pmatrix} \] 
(8b)

These are the normal equations used in Wiener single channel filter design and can be solved rapidly using Levinson’s recursion algorithm.

**Examples**

Here we use the Ex (electric field parallel to north-south) component of MT time series in one of the measured sites which was located in 3 km from the huge power line, for testing the algorithm. It is important to note that our measurement was carried out in 8-256 Hz frequency range. After primarily corrections on this electrical component field we divided Ex time series to 110 time segments then stacked all of them. So the results of stacked primary input time series, notch filtered and proposed algorithm outputs are shown below (figure 1).
CONCLUSIONS

By comparison between amplitude spectrum resulted from notch filtering and wiener algorithm noise suppression which illustrated in figure 2, we can infer that in notch filtering, with suppression the contaminating sinusoidal noise, the signal frequencies around the power line frequency are also attenuated, while in our algorithm without attenuating the signals, the mentioned noise can be suppressed.

Figure 1. Stacked primary input time series (upper panel), notch filtered (middle panel) and proposed algorithm outputs (lower panel).
REFERENCES


