COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS UNDER UNIFORM LOADS OF PLATES WITH HINGED SUPPORT

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ABSTRACT : Plates with hinged supports on four edges often constitute structural systems. These plates are used in the box-section columns and I section beams.

If these plates are forced on two opposite edges with uniform distributed load, they tend to crease before the material fails.

Plates with a side ratio of $\alpha \geq 1$ are calculated with SAP2000 structural analysis software and they will compare to former results of experimentally investigated research, the causes and results will present the end of the article.

INTRODUCTION

In plates with hinged supports on four edges exposed to pressure stretching the first deformation before steel material loses its endurance is the local buckling behaviour with the influence of pressure stretching applied from reciprocal sides. [7]

Buckling happens at plates exposed to pressure stretching from reciprocal sides. The calculation of the buckling weight of the plate with hinged supports on four edges when it is exposed to smooth spreaded pressure stretching from the reciprocal sides is desired.

A plate with hinged supports on four edges is defined in Sap2000 software and the plate's buckling stretching calculation will be done when the plate is pressured from two sides.

Buckling weights of this kind of plates were first theoretically calculated in 1891 in England by G. H. BRYAN.[1]

The calculation of plate buckling with theoretical calculation methods in literature was examined empirically at the resource no [1] and results are given as tables. The experiment plates with different dimensions mentioned in the subjected resource are solved with Sap2000 software and the laboratory results and the results in Sap2000 were compared.

PLATE BUCKLING

Because plates are delicate materials, they buckle perpendicularly to the pressure plane when they are pressured with pressure loads under a certain load. [1]

The plates' buckling behaviour depends on different parameters. Some parameters will be introduced from linear and anti linear buckling theories.

Plate delicateness is defined as the proportion of plate width b and plate thickness t. Plate width b is the dimension of the plate side loaded with pressure. In studies, the "b/t" proportion varies within the range of 66,67 and 133,33 in plates with 3 mm thickness and within the range of 50 and 100 in plates with 4 mm thickness.

Plate side proportion is

$$\alpha = a / b \tag{2.1}$$

and it is the proportion of plate length to width. In the literature [1], experiments were repeated with the plates with different "α" proportions (Figure 1).

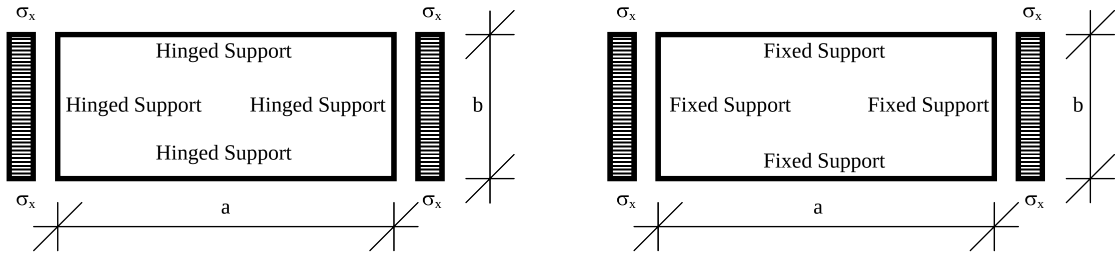

Figure 1. Plate and Load

Influence of side support in width to k buckling coefficient, buckling coefficient at plates with hinged supports on four edges is calculated with;

$$k = \left[\frac{m}{\alpha} + \frac{n^2}{m} \alpha \right] \tag{2.2}$$

formula[1].

DEFINING THE PLATE BUCKLING IN SAP2000

When a plate with hinged supports on four edges is exposed to pressure stretching from reciprocal two sides, it buckles at value σ_{cr}. This σ_{cr} statement can be calculated with the formula given in Omer W. Bladgett's book.

In studies, in resource no [1], plates with t = 3 mm and t = 4 mm thickness were used as experiment samples and the buckling stretching of these plates were calculated empirically; the same samples will be defined in SAP2000 software. From "St 37" steel in a single piece, these plates' dimensions are 400 x 400, 400 x 350, 400 x 300, 400 x 250 ve 400 x 200.

Coefficient values and buckling stretching which will be calculated from several calculations are independent of pressure stretching values applied to the plates.

The above mentioned plate dimensions of 3 mm and 4 mm thickness' were defined one by one as plates with hinged supports on four edges in SAP2000 software and these plates were subjected to a smooth separated pressure load with a value of 110 kN from two sides. The buckling stretching value of the plates is calculated as a coefficient factor calculated with the software.

The coefficient factor calculated with the software gives the stretching value which will buckle at the linear elastic limit of the plate at an instance value of the coefficient factor of pressure loading values of the samples subjected to loading in the software.

In experiments done in resource no [1], calculations were repeated two times for each different sample, i.e. for 10 different samples, 20 calculations were done. However, because it is not possible to have different results under the same conditions, one result is received for each sample. Multiplying the applied pressure strength with the plate's factor coefficient can be learned from the results, P_{cr} load is calculated. This value expresses the critic local buckling load according to the plate's elastic calculation. Buckling stretching σ_{cr} (kN/cm^2) at the plate's elastic limit is calculated by dividing the calculated P_{cr} elastic buckling value with A = b x t width cross-section area.

σ_D / σ_F calculated in experiments done in resource no [1], is calculated under laboratory conditions as a coefficient value without a unit as the proportion of plate's losing its load strength σ_D value to σ_F. Here σ_F is the flow value of the steel used in the experiment because the used material is "St37" steel σ_F = 24,87 kN / cm^2.

In order to compare the σ_D / σ_F value calculated in the above mentioned experiments with the values calculated with SAP2000 software, by dividing the plate's buckling stretching at elastic zone σ_{cr} with steel's flow stretching, σ_{cr} / σ_F coefficient is calculated as a comparable expression.

Table 3.1 : Sample Cross-section Values [1]

Sample No	Plate Height (mm)	Plate Width (mm)	Plate Thic. (mm)	α Side Ratio	b/t
S1	400	400	3	1.00	133.33
S2	400	400	3	1.00	133.33
S3	400	350	3	1.14	116.67
S4	400	350	3	1.14	116.67
S5	400	300	3	1.33	100.00
S6	400	300	3	1.33	100.00
S7	400	250	3	1.60	83.33
S8	400	250	3	1.60	83.33
S9	400	200	3	2.00	66.67
S10	400	200	3	2.00	66.67
S11	400	400	4	1.00	100.00
S12	400	400	4	1.00	100.00
S13	400	350	4	1.14	87.50
S14	400	350	4	1.14	87.50
S15	400	300	4	1.33	75.00
S16	400	300	4	1.33	75.00
S17	400	250	4	1.60	62.50
S18	400	250	4	1.60	62.50
S19	400	200	4	2.00	50.00

S20	400	200	4	2.00	50.00

EXAMINATION OF EMPRICAL AND THEORETICAL RESULTS OF BUCKLING STRETCHING

While the plates' buckling are examined, regarding the experimentally used samples' material, support conditions and cross-section features, the same values are defined in SAP2000 and the obtained results were significantly different from the empirical results. In the parallel direction to the plates' reciprocal sides' pressure loads in the areas near the supports, local buckling occurs in the middle parts of the plate before it loses its load strength. With formulation or SAP2000 software, σ_{cr} the stretching value is calculated as buckling expression at the linear elastic limit. At this stretching value, in the effective width areas near the supports, the load strength of the plate has not disappeared yet. Therefore, the theoretically calculated value is the buckling value at the linear elastic limit. [7,8,2]

Under laboratory conditions, with the same material and cross-section feature components supplied to be loaded at the plastic limit, i.e. load strength limit, the plate's buckling value at plastic limit is obtained. If results obtained empirically are compared with the results calculated with SAP2000, it is derived that the plates' buckling loads are higher in the empirical results. As mentioned above, as a plate's load strength is entirely lost, the stretching value is found in the experiments and the comparison results seem to be meaningful. The most significant design criterion in thin plates is the buckling feature. [9]

After transactions are done, the σ_{cr} / σ_F value is calculated and σ_D / σ_F values obtained empirically from resource no [1] beforehand are given comparatively in the table.

In the graphics below, values obtained empirically and values calculated with SAP2000 will be shown in diagram.

In addition, while examining the plate buckling values theoretically, in Omer W. Bladgett's Design of Welded Structures, the numerical example relevant to the calculation of hinged and supported plates' buckling stretching value is adjusted to SAP2000 and results are meaningfully approximate with the results in the above mentioned resource.

Example: $\sigma_F = 23.20$ kN / cm^2 , b = 50.80 cm, t = 0.64 cm k = 4

$$b / t = 50.80 / 0.64 = 80$$

Values take place in the expression

$$\frac{b/t}{\sqrt{k}} \tag{4.1}$$

$$\frac{b/t}{\sqrt{k}} = \frac{50.80/0.64}{\sqrt{4}} = 40$$

is calculated.

$$\sigma_{cr} = \left[\frac{117.58}{(b/t)/\sqrt{k}} \right]^2 = [117.58/40]^2 = 8.64 kN / cm^2$$

While defining the same support conditions with same units, material and cross-section features in SAP2000, calculated under given pressure load, and; σ_{cr} = 9.33 kN / cm^2 is calculated.

Here, with an 8 % proximity the same result is obtained. This value is the local buckling value and it gives the buckling stretching at linear elastic limit.

PROCESS DIFFERENCES IN CALCULATION OF BUCKLING STRETCHING WITH EMPIRICIAL METHOD AND SAP2000 SOFTWARE

When a plate with hinged supports on four edges is exposed to pressure effect, in reaching empirical results to calculate buckling stretching occurring at the plate and in calculation theoretically, one or more of the calculation methods given below are used. [13]

Traditional calculation method, i.e. linear elastic method;

F = k x u (5.1)

F: strength ; k : rigidity ; u : replacement.

Anti linear calculation method

Large deformation effect

F ≠ k x u (Large Deformation) (5.2)

P – Δ effect (5.3)

M = F x h + P x δ (5.4)

Systems which are not linear on account of materials.

If plates’ buckling stretching is found under laboratory conditions;

P_D : Plates’ collapse strength, i.e. buckling strength includes;

Anti linear features on account of material

P – Δ effect feature.

In this case, the plate exposed to pressure stretching from its two sides, beyond directional elastic limit, indicating a plastic behaviour deforms i.e. buckles under P_D load which is the load that the plate loses its bearing strength. The Poisson Ratio is 0.3 in the calculations. [10]

In SAP2000 software, the plate is modeled with a shell material model and it is analysed according to the P – Δ effect and the large deformation effect 2a and 2b which are of anti linear calculation methods. In the calculation of wrinkling (Local Buckling), in "Buckling Load" it is calculated only with Euler Buckling stretching.

Therefore, P_D found under laboratory conditions gives the pressure load when the plate is buckled on the plastic limit; and σ_D gives the buckling stretching beyond the plate's linear elastic limit. [12]

In the calculation done with SAP2000 software, parallel to the theoretical calculation, it gives the stretching value at the linear elastic limit. If the plate's delicateness increases or decreases, it is learned that empirical results' and theoretical results' change coherently.

CALCULATION OF PLATE BUCKLING STRETCHING WITH SAP2000 SOFTWARE

In literature [1] for samples E1 and E2 subjected to experiments calculations will be shown and for other samples, derived results will be given in tables.

Plate dimensions: a = 400 mm , b = 400 mm , t = 3 mm , P =100 kN , E = 21000 kN / cm^2

Coefficient Factor Obtained In Sap2000;

SF: 0,398 P_{cr} = 100 x 0,398 P_{cr} = 39.80 kN

SF value is a coefficient that is derived by the software and it shows that the plate will wrinkle on the plate's elastic limit with 0.398 times worth of exposed pressure load.

Width Cross-Section Area Of The Plate;

A = 400 x 3 = 12 cm^2 .

$\sigma_{cr} = P_{cr} / A$ [2] (6.1)

σ_{cr} = 39.8 / 12 = 3,316 kN / cm^2 .

In Order To Compare With The Coefficients In Literature [1];

σ_{cr} / σ_F value is needed.

Here σ_F = 24.87 kN / cm^2 and it expresses the "St 37" type steel's flow stretching.

σ_{cr} / σ_F = 3,316 / 24,87 = 0,133

coefficient is found

In The Empirical Calculation;

σ_D / σ_F = 0,236 ve 0,288 values are found.

σ_{Tcr} = k x σ_e = 7.592.10^4 x (t/b)^2 (6.2)

σ_{Tcr} = 7.592.10^4 x (3/400)^2 = 4.207 kN / cm^2

σ_D stretching values of empirical results and σ_D stretching found with SAP2000 and σ_D theoretical stretching values calculated with (6.2) correlation comparatively are given in the tables.

Here, the empirical result gives the stretching when the plate loses its veering strength the theoretical result gives the elastic stretching value at the linear elastic limit when the plate had not lost its bearing strength yet.

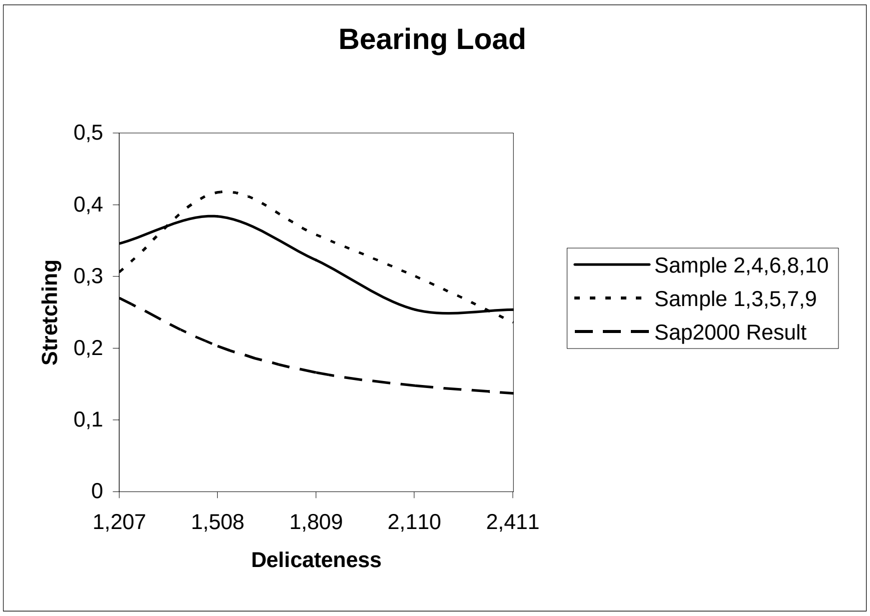

Figure 6.1 Bearing Load Delicateness Diagram (t = 3 mm)

Table 6.1. Comparison of the results obtained with SAP 2000 and the empirical results in literature [1]

Sample No	α Side Ratio	b/t	λ_V Plate Delic.	Empiric σ_D / σ_F	Sap2000 σ_{cr} / σ_F
S1	1.00	133.33	2.413	0.236	0.133
S2	1.00	133.33	2.413	0.288	″
S3	1.14	116.67	2.112	0.300	0.145
S4	1.14	116.67	2.112	0.254	″
S5	1.33	100.00	1.809	0.358	0.164
S6	1.33	100.00	1.809	0.323	″
S7	1.60	83.33	1.508	0.417	0.194
S8	1.60	83.33	1.508	0.384	″
S9	2.00	66.67	1.207	0.306	0.246
S10	2.00	66.67	1.207	0.346	″
S11	1.00	100.00	1.809	0.430	0.237
S12	1.00	100.00	1.809	0.471	″
S13	1.14	87.50	1.584	0.432	0.258
S14	1.14	87.50	1.584	0.442	″
S15	1.33	75.00	1.357	0.530	0.291
S16	1.33	75.00	1.357	0.485	″
S17	1.60	62.50	1.131	0.571	0.345
S18	1.60	62.50	1.131	0.609	″
S19	2.00	50.00	0.905	0.525	0.438
S20	2.00	50.00	0.905	0.568	″

Table-6.2. Comparison of theoretical results with the results obtained with SAP 2000 and the empirical results in literature [1]

Sample No	α Side Ratio	b/t	Empiric σ_D	Sap2000 σ_{cr}	Theor. σ_{Tcr}
S1	1.00	133.33	5.870	3.316	4.270
S2	1.00	133.33	7.163	3.316	4.270
S3	1.14	116.67	7.461	3.600	5.577
S4	1.14	116.67	6.317	3.600	5.577
S5	1.33	100.00	8.903	4.067	7.592
S6	1.33	100.00	8.033	4.067	7.592
S7	1.60	83.33	10.371	4.825	10.933
S8	1.60	83.33	9.550	4.825	10.933
S9	2.00	66.67	7.610	6.125	17.080
S10	2.00	66.67	8.605	6.125	17.080
S11	1.00	100.00	10.694	5.900	7.592
S12	1.00	100.00	11.714	5.900	7.592
S13	1.14	87.50	10.744	6.406	9.916
S14	1.14	87.50	10.993	6.406	9.916
S15	1.33	75.00	13.181	7.225	13.496
S16	1.33	75.00	12.062	7.225	13.496
S17	1.60	62.50	14.200	8.581	19.436
S18	1.60	62.50	15.146	8.581	19.436

S19	2.00	50.00	13.057	10.880	30.368
S20	2.00	50.00	14.126	10.880	30.368

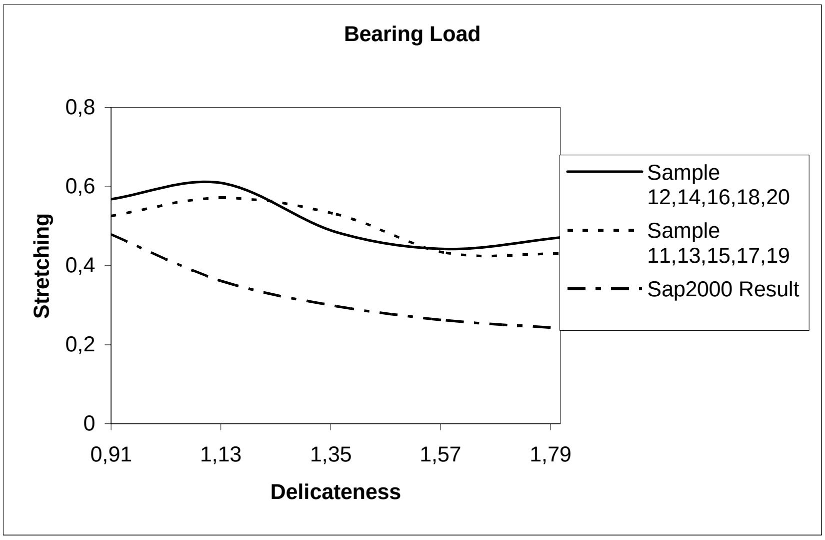

Figure-6.2. Bearing Load λ_V Delicateness Diagram (t = 4 mm)

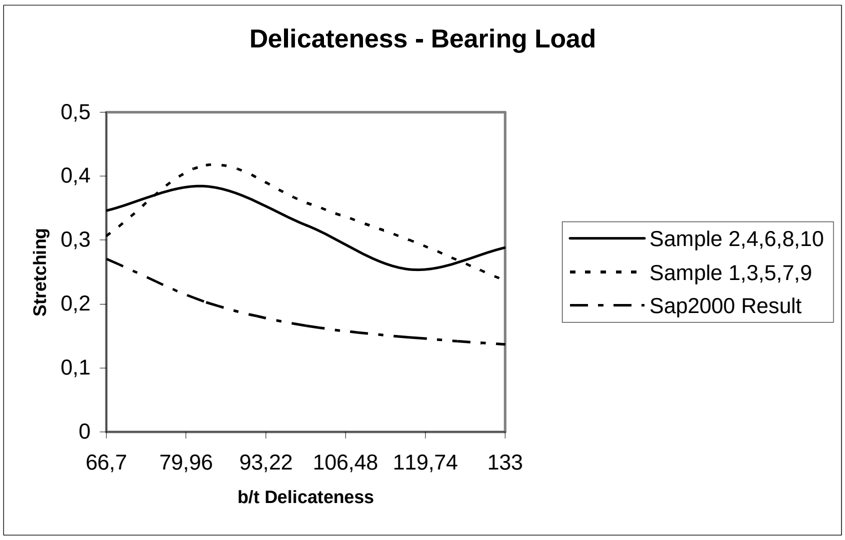

Figure-6.3. Bearing Load - b/t Delicateness Diagram (t = 3 mm)

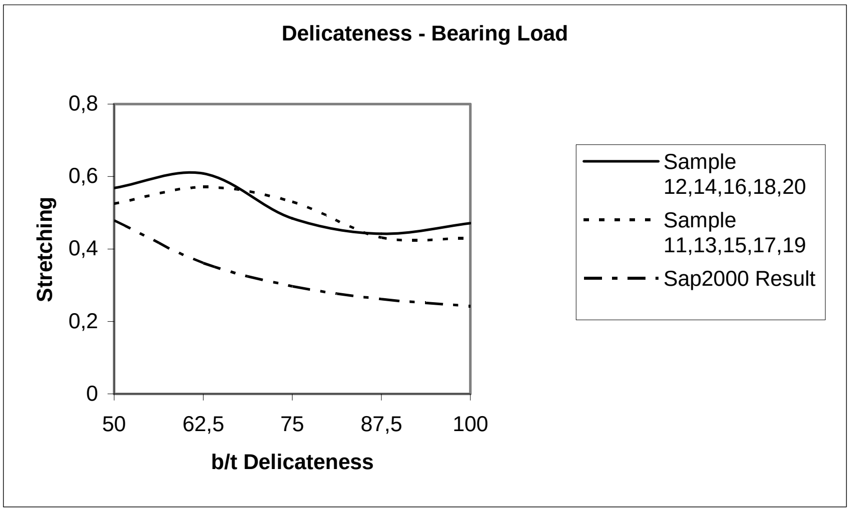

Figure-6.4. Bearing Load - b/t Delicateness Diagram (t = 4 mm)

CONSLUSION AND SUGGESTIONS

Experiments done in literature [1] and stretching values calculated with SAP2000 are compared and it is learned that the results in SAP2000 are lower than the empirical results. That difference is the difference between the value reaching the wrinkling stretching at the plastic limit where the plate loses its bearing strength entirely under laboratory conditions and the wrinkling stretching at the elastic limit calculated with SAP2000.

Elastic wrinkling stretching values are learned with theoretical calculation and by SAP2000 overlap with an proximity of 8%.

At box and U cross-sectioned joists, in order to prevent buckling, rigiding components (Reinforce Plate) should be used [11]..

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