INTERNATIONAL EARTHQUAKE SYMPOSIUM KOCAELI 2007
22 - 26 OCTOBER 2007

FREQUENCY OPTIMIZATION OF FUNCTIONALLY GRADED LAMINATED CYLINDRICAL SHELLS

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Key Words: Functionally graded materials, laminated cylindrical shell, optimization, frequency

ABSTRACT

We used the first order shear deformation theory (FSDT) coupled with the finite element method (FEM) to study free vibrations of simply supported functionally graded (FG) anisotropic laminated cylindrical shell with the objective of maximizing of its natural frequencies. The design variable is the p that maximizes the objective function. We assume that a FG anisotropic shell in which the fiber orientations vary smoothly through shell thickness. The parametric studies are performed to investigate the effect of shell aspect ratio on the optimal designs and the results are compared.

INTRODUCTION

In conventional laminated composite structures, homogeneous elastic laminae are bonded together to obtain enhanced mechanical properties. However, the abrupt change in material properties across the interface between different materials can result in large interlaminar stresses leading to delamination. Furthermore, large plastic deformations at the interface may trigger the initiation and propagation of cracks in the material. One way to overcome these adverse effects is to use “functionally graded materials” in which material properties vary continuously. This is achieved either by gradually changing the volume fraction of the constituent materials, usually in the thickness direction only, or by changing the chemical structure of a thin polymer sheet to obtain a smooth variation of in-plane material properties and an optimum response to external thermomechanical loads. We assume that a FG anisotropic shell in which the fiber orientations vary smoothly through shell thickness.

Several studies have been performed to analyze the frequency behavior of functionally graded shells. Zhi-yuan and Hua-ning, (2007) studied free vibration analysis of functionally graded material (FGM) cylindrical shells with holes. Non-dimensional frequencies of shell with holes of different shape, number, location were given. Haddadpour *et al.,* (2007) performed free vibration analysis of simply supported FG cylindrical shells for four sets of in-plane boundary conditions. The equations of motion were based on Love’s shell theory and the von Karman–Donnell-type of kinematic nonlinearity. Shakeri *et al.*, (2006) presented the analysis of functionally graded thick hollow cylinders under dynamic load. Material’s properties in each layer were constant and functionally graded properties were resulted by suitable arrangement of layers in multilayer cylinder. The Navier equation was solved by Galerkin finite element and Newmark methods. Bhangale *et al.,* (2006) used a finite element formulation based on First-Order Shear Deformation Theory (FSDT) to study the thermal buckling and vibration behavior of truncated FGM conical shells in a high-temperature environment. The material properties of the truncated FGM conical shells were functionally graded in the thickness direction according to a volume fraction power law distribution. Kadoli and Ganesan, (2006) presented linear thermal buckling and free vibration analysis for functionally graded cylindrical shells with clamped–clamped boundary condition based on temperature-dependent material properties. Patel *et al., (2005) analyzed* the free vibration characteristics of functionally graded elliptical cylindrical shells using finite element formulated based on the theory with higher-order through the thickness approximations of both in-plane and transverse displacements. The detailed parametric studies were carried out to study the influences of non-circularity, radius-to-thickness ratio, material composition and material profile index on the free vibration frequencies and mode shape characteristics of functionally graded elliptical shells.

BASIC EQUATIONS

Consider a fiber-reinforced laminated composite circular cylindrical shell of finite length L, radius R, total wall thickness h ($h=\sum h_i$, h_i represents thickness of a layer) and composed of finite number, N, of uniform-thickness orthotropic layers as shown in Figure 1. The cylindrical coordinates (x, θ, z) are so chosen so that z and x are the axial and radial coordinates, respectively. Based on the first order shear deformation theory, inplane and transverse displacements are assumed in the following form:

$$u(x,\theta,z,t)=u_o(x,\theta,t)+z\phi_x(x,\theta,t)$$

$$v(x,\theta,z,t)=v_o(x,\theta,t)+z\phi_\theta(x,\theta,t) \tag{1}$$

$$w(x,\theta,z,t)=w_o(x,\theta,t)$$

Here, u_o, v_o, w_o reply for displacements of a point in the mid or reference surface in longitudinal, circumferential, and transverse directions, respectively; ϕ_x and ϕ_θ are the rotations of normal to the mid-surface about y and x axes, respectively.

Based on Hamilton's principle, the equations of motion in terms of the forces and moment resultants can be written as:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{R\partial\theta}=I_o\ddot{u}_o+I_1\ddot{\phi}_x$$

$$\frac{\partial N_{x\theta}}{\partial x}+\frac{\partial N_\theta}{R\partial\theta}=I_o\ddot{v}_o+I_1\ddot{\phi}_\theta$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\theta}{R\partial\theta}-\frac{N_\theta}{R}=I_o\ddot{u}_o+I_1\ddot{w}_o \tag{2}$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{\theta x}}{R\partial\theta}-Q_x=I_1\ddot{u}_o+I_2\ddot{\phi}_\theta$$

$$\frac{\partial M_{x\theta}}{\partial x}+\frac{\partial M_\theta}{R\partial\theta}+Q_x=I_1\ddot{v}_o+I_2\ddot{\phi}_\theta$$

The in-plane stress resultants $N_x, N_\theta, N_{x\theta}, N_{\theta x}$, stress-couple resultants $M_x, M_\theta, M_{x\theta}, M_{\theta x}$ and the transverse stress resultants Q_x, Q_θ in the laminated shell are

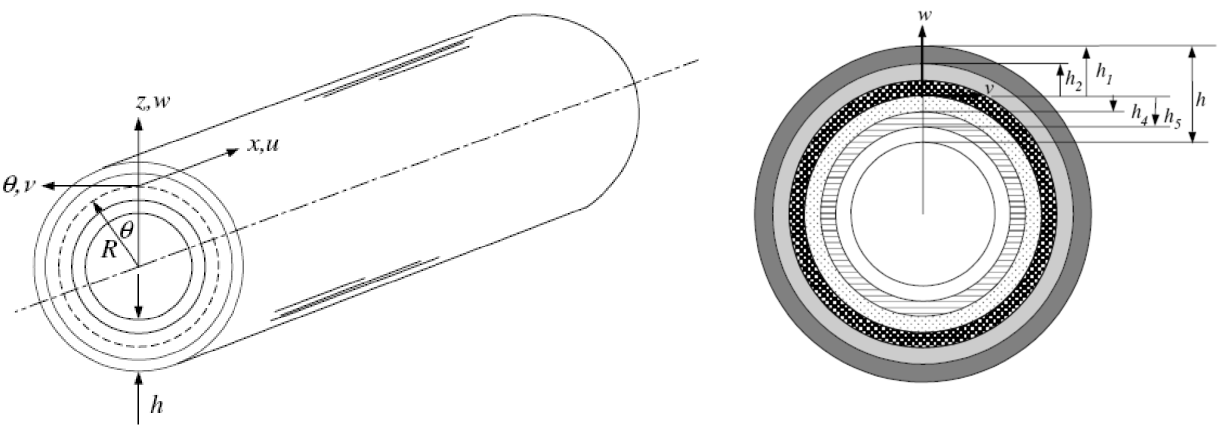
$$\{N_x,N_\theta,N_{x\theta},N_{\theta x}\}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\{\sigma_x(1+z/R),\sigma_\theta,\sigma_{x\theta}(1+z/R),\sigma_x\}dz$$

$$\{M_x,M_\theta,M_{x\theta},M_{\theta x}\}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\{\sigma_x(1+z/R),\sigma_\theta,\sigma_{x\theta}(1+z/R),\sigma_x\}zdz \tag{3}$$

$$\left\{Q_x, Q_\theta\right\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left\{\sigma_{xz}(1+z/R), \sigma_{\theta z}\right\} dz$$

and the mass moments of inertia are

$$\left(I_o, I_1, I_2\right) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^k (1, z, z^2) dz \tag{4}$$



(a) (b)

Figure-1. Geometry of cylindrical laminated shell (a) and cross-sectional view (b)

For orthotropic layers, the compliance matrix and stress–strain relation in the material coordinates are of the form

$$\left[S_{ij}\right] = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{21}/E_2 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_2 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \tag{6}$$

The coordinate system used in the solution of a problem does not coincide with the material coordinate system. In addition, this composite laminated shell has several layers, each with different orientation. Thus, we need to establish a transformation relation:

$$\left[\bar{Q}\right]=[T][Q][T]^{-T} \tag{7}$$

where

$$[T]=\begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 0 & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 0 & 0 & 2\sin\theta\cos\theta \\ 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & -\sin\theta & \cos\theta & 0 \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & 0 & 0 & \cos^2\theta-\sin^2\theta \end{bmatrix} \tag{8}$$

Now we obtain the stress–strain relations in the problem coordinates:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{\theta z} \\ \sigma_{xz} \\ \sigma_{x\theta} \end{Bmatrix}=\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & \bar{Q}_{26} \\ 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \tag{9}$$

The strains for a shell are defined as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} \end{Bmatrix}=\begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{R\partial\theta}+\dfrac{w}{R} \\ \dfrac{\partial v}{\partial z}+\dfrac{\partial w}{R\partial\theta} \\ \dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x} \\ \dfrac{\partial v}{\partial x}+\dfrac{\partial u}{R\partial\theta} \end{Bmatrix} \tag{10}$$

By using the first-order theory and substituting Eq. (1) into Eq. (10), we obtain

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} \end{Bmatrix}=\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_\theta^o \\ \gamma_{\theta z}^o \\ \gamma_{xz}^o \\ \gamma_{x\theta}^o \end{Bmatrix}+z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_\theta^1 \\ \gamma_{\theta z}^1 \\ \gamma_{xz}^1 \\ \gamma_{x\theta}^1 \end{Bmatrix} \tag{11}$$

or

$$\{\varepsilon\} = \{\varepsilon^o\} + z\{\varepsilon^1\} \tag{12}$$

where

$$\{\varepsilon^o\} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_\theta^o \\ \gamma_{\theta z}^o \\ \gamma_{xz}^o \\ \gamma_{x\theta}^o \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{R\partial\theta} + \dfrac{w_o}{R} \\ \dfrac{\partial v_o}{\partial z} + \dfrac{\partial w_o}{R\partial\theta} \\ \dfrac{\partial u_o}{\partial z} + \dfrac{\partial w_o}{\partial x} \\ \dfrac{\partial v_o}{\partial x} + \dfrac{\partial u_o}{R\partial\theta} \end{Bmatrix} \tag{13}$$

$$\{\varepsilon^1\} = \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_\theta^1 \\ \gamma_{\theta z}^1 \\ \gamma_{xz}^1 \\ \gamma_{x\theta}^1 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_\theta}{R\partial\theta} \\ 0 \\ 0 \\ \dfrac{\partial \phi_\theta}{\partial x} + \dfrac{\partial \phi_x}{R\partial\theta} \end{Bmatrix} \tag{14}$$

By substituting Eqs. (9), (11), (13) and (14) into Eq. (3) the force and moment resultants can be obtained as:

$$\begin{aligned} N_x = {} & A_{11}\frac{\partial u_o}{\partial x} + A_{12}\left(\frac{w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right) + A_{16}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right) \\ & + \frac{1}{R}D_{11}\frac{\partial \phi_x}{\partial x} + \frac{1}{R}D_{12}\frac{\partial \phi_\theta}{\partial\theta} + \frac{1}{R}D_{16}\left(\frac{\partial \phi_\theta}{\partial x} + \frac{\partial \phi_x}{R\partial\theta}\right) \end{aligned}$$

$$N_\theta = A_{12}\frac{\partial u_o}{\partial x} + A_{22}\left(\frac{w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right) + A_{26}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right)$$

$$\begin{aligned} N_{x\theta} = {} & A_{16}\frac{\partial u_o}{\partial x} + A_{26}\left(\frac{w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right) + A_{66}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right) \\ & + \frac{1}{R}D_{66}\left(\frac{\partial \phi_\theta}{\partial x} + \frac{\partial \phi_x}{R\partial\theta}\right) + \frac{1}{R}D_{16}\frac{\partial \phi_x}{\partial x} + \frac{1}{R}D_{26}\frac{\partial \phi_\theta}{\partial\theta} \end{aligned}$$

$$N_{\theta x} = A_{16}\frac{\partial u_o}{\partial x} + A_{26}\left(\frac{w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right) + A_{66}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right)$$

$$M_x = D_{11}\frac{\partial\phi_x}{\partial x} + \frac{1}{R}D_{12}\frac{\partial\phi_\theta}{\partial\theta} + \frac{1}{R}D_{11}\frac{\partial u_o}{\partial x} + \frac{1}{R}D_{12}\left(\frac{\partial w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right) + \frac{1}{R}D_{16}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right) + D_{16}\left(\frac{\partial\phi_\theta}{\partial x} + \frac{\partial\phi_x}{R\partial\theta}\right)$$

$$M_\theta = D_{12}\frac{\partial\phi_x}{\partial x} + \frac{1}{R}D_{22}\frac{\partial\phi_\theta}{\partial\theta} + D_{26}\left(\frac{\partial\phi_\theta}{\partial x} + \frac{\partial\phi_X}{R\partial\theta}\right)$$

$$M_{x\theta} = D_{66}\left(\frac{\partial\phi_\theta}{\partial x} + \frac{\partial\phi_x}{R\partial\theta}\right) + D_{16}\frac{\partial\phi_x}{\partial x} + \frac{1}{R}D_{26}\frac{\partial\phi_\theta}{\partial\theta} + \frac{1}{R}D_{66}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{R\partial\theta}\right) + \frac{1}{R}D_{16}\frac{\partial u_o}{\partial x} + \frac{1}{R}D_{26}\left(\frac{w_o}{R} + \frac{\partial v_o}{R\partial\theta}\right)$$

$$M_{\theta x} = D_{66}\left(\frac{\partial\phi_\theta}{\partial x} + \frac{\partial\phi_x}{R\partial\theta}\right) + D_{16}\frac{\partial\phi_x}{\partial x} + D_{26}\frac{\partial\phi_\theta}{R\partial\theta}$$

$$Q_x = KA_{45}\left(\phi_\theta + \frac{\partial w_o}{R\partial\theta}\right) + A_{55}\left(\phi_x + \frac{\partial w_o}{\partial x}\right)$$

$$Q_y = KA_{44}\left(\phi_\theta + \frac{\partial w_o}{R\partial\theta}\right) + A_{45}\left(\phi_x + \frac{\partial w_o}{\partial x}\right) \tag{15}$$

where the extension and bending stiffness are defined as:

$$A_{ij} = \sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\bar{Q}_{ij}dz \tag{16}$$

$$D_{ij} = \sum_{k=1}^{N}\int_{z_{k+1}}^{z_k}\bar{Q}_{ij}z^2dz \tag{17}$$

where z_k and z_{k+1} denote, respectively, the distances of the kth layer and k+1th layer from the shell reference surface.

FINITE ELEMENT FORMULATION

In this study, the C°-continuity element with fives degrees of u_o, v_o, w_o, ϕ_x and ϕ_y is used. Interpolation functions can be assumed as

$$u_o^e = \sum_{i=1}^{N_n} u_i^e N_i^e, v_o^e = \sum_{i=1}^{N_n} v_i^e N_i^e, w_o^e = \sum_{i=1}^{N_n} w_i^e N_i^e \tag{18}$$

$$\phi_x^e = \sum_{i=1}^{N_n} \psi_{xi}^e N_i^e, \phi_y^e = \sum_{i=1}^{N_n} \psi_{yi}^e N_i^e$$

where N_i represents the element interpolation functions and N_n is the number of nodes per element. In this study, the four-node Lagrangian finite element approach is used for the analysis of cylindrical laminated shells. When the FEA based on the Reissner-Mindlin shell theory is applied to thin shells, the shear locking may occur. Reduced/selective integration technique is adopted for the element matrices in order to avoid possible shear locking. The stiffness matrix of the shell is obtained by using the minimum potential energy principle. The kinetic and strain energy of the shell can be found as

$$T = \frac{1}{2}\iint \left[\sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \rho_k \left\{ \dot{u}^k \dot{v}^k \dot{w}^k \right\} x \left\{ \dot{u}^k \dot{v}^k \dot{w}^k \right\}^T \left(1 + \frac{z}{R}\right) dz \right] dxdy \tag{19}$$

$$U = \frac{1}{2}\iint \left[\left\{ \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \{\sigma\}^T \{\varepsilon\} \right\} \left(1 + \frac{z}{R}\right) dz \right] dxdy \tag{20}$$

where ρ_k and $\left\{ \dot{u}^k \dot{v}^k \dot{w}^k \right\}$ are denstiy and velocity vector of the kth layer, respectively. Applying Lagrangian equations of motion:

$$d/dt\left[\partial(T-U)/\partial\dot{\delta}_i\right] - \left[\partial(T-U)/\partial\delta_i\right] = 0 \qquad \text{i=1: N} \tag{21}$$

gives the element governing equations of motion as follows:

$$\left[M^e\right]\left\{\ddot{\delta}^e\right\} + \left[K^e\right]\left\{\delta^e\right\} = \{0\} \tag{22}$$

where $\left[M^e\right]$ and $\left[K^e\right]$ are element mass and stiffness matrices, respectively. After constructing the element matrices, through the finite elements assembly procedure, the global finite element matrices are found to form the finite element equation, the free vibration problem of the shell becomes as follows:

$$\left[[K]^G + \omega^2 [M]^G\right]\{\delta\}^G = \{0\} \qquad (23)$$

which is an eigenvalue problem of which the solution with an appropriate method will give the natural frequencies (ω) of the laminated cylindrical shell. In this study, subspace iteration technique is used for frequency analysis of shells.

FONCTIONALLY GRADED MATERIAL

The desired gradation of material properties in the thickness direction is attained by varying the fiber orientation angle, y; in a graphite/epoxy plate according to the relation

$$\theta = \frac{\pi}{2}\left(\frac{1}{2} + \frac{z}{h}\right)^p, \quad -\frac{h}{2} \le z \le \frac{h}{2} \qquad (24)$$

Thus, the fiber orientation angle varies continuously from 0° at the bottom surface to 90° at the top surface; For different values of p, Fig. 2 exhibits the variation of the fiber orientation through the shell thickness. This can be achieved experimentally by laying successively thin layers of different fiber orientations. Even though we have considered integer valuesof p, it could be any real number. Whereas the gradient of the fiber orientation is continuous for p>0; it may be discontinuous for p<0.

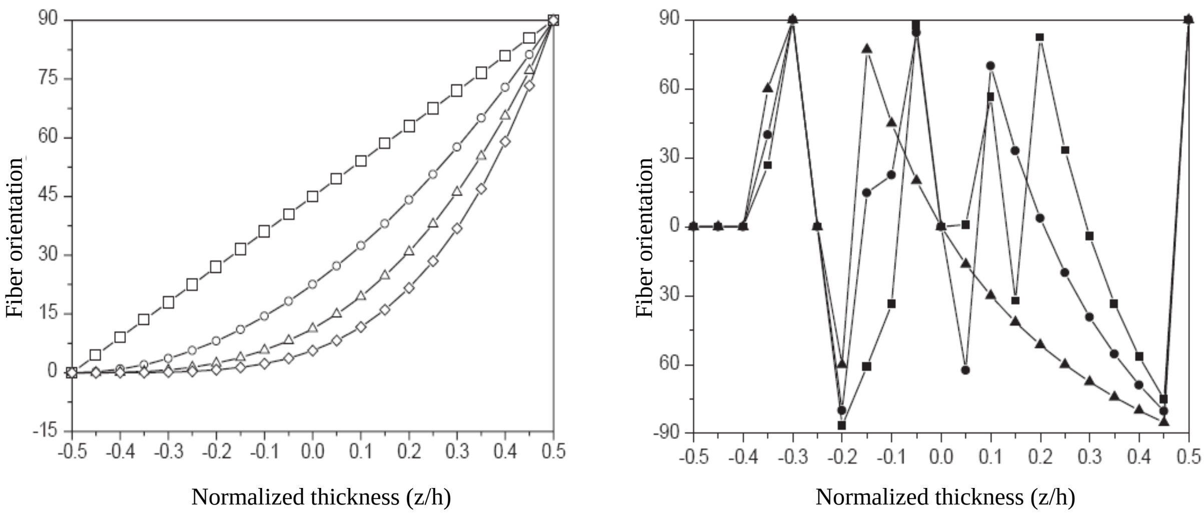


Figure-2. For different values of the exponent p, variation of the fiber orientation through the plate thickness: (a) ▲, p=-1; ●, p=-2, ■, p=-3; (b) □ , p=1, o, p=2; Δ , p=3; ◊, p=4

OPTIMIZATION PROBLEM

An objectİve function is taken to be ω for the fundamental mode of the laminated shell. The term ‘fundamental frequency’ indicates the lowest eigenvalue for given conditions. A vibration mode is identified by a pair of integers (m,n) where m and n are the half wavenumbers in assumed solutions. It is noted in the present problem that the fundamental mode is not limited to (m,n)=(1,1) and may take on other wavenumbers, such as (1,2) or (2,1). This tendency is not unusual in the vibration and buckling problems of shells due to the effect of geometric curvatures and is more vividly seen in closed cylindrical shells.

The mathematical expression for such an optimization problem is written as

$$\omega_1 = \omega_1(p) \rightarrow \text{Max (objective function)}$$

$$p \text{ (design variable)} \qquad (25)$$

$$-3 \le p \le 4$$

NUMERICAL RESULTS AND DISCUSSION

For the computations, a typical T300/5208 graphite/epoxy material plate with the following properties is used: $E_1 = 181\text{GPa}$, $E_2 = 10.3\text{GPa}$, $G_{12} = 7.17\text{GPa}$, ν_{12} =0.28, ρ =1600 kg/m^3. Laminated cylindrical shell is constructed of equal thickness layers and H/R=0.2 is considered.

Effect of shell aspect ratio (L/R)

Results presented below have been obtained by dividing the shell thickness into 20 uniform layers with the fiber orientation in a layer computed from Eq. (24) by setting z equal to the coordinate of a point on the midsurface of the layer. Results included in Table 1, clearly establish that the frequency of a laminated shell can be maximized by smoothly changing the fiber-orientation through the shell thickness. The value of p that maximizes the frequency can be determined by plotting the frequency vs. p and estimating the value of p. The effect of shell aspect ratio on the optimal designs is investigated for L/R=1, L/R=2, L/R=4 and L/R=6. As seen, the fundamental frequency is obtained for p=-3 for all L/R ratios. As expected, the fundamental frequency decreases with increase in the L/R ratio. The differences for fundamental frequencies are 48.91%, 45.18% and 22.20% between L/R=1-2, L/R=2-4 and L/R=4-6 laminates. As seen, as L/R ratio increases the differences for fundamental frequencies decrease.

Table-1. Effect of L/R ratios on the optimal design for laminated cylindrical shell

	ω (cycles/sec)			
	L/R=1	L/R=2	L/R=4	L/R=6
p=1	$3.87182\text{x}10^3$	$1.7926\text{x}10^3$	$9.06494\text{x}10^2$	$6.62862\text{x}10^2$
p=2	$3.89112\text{x}10^3$	$1.82503\text{x}10^3$	$9.35446\text{x}10^2$	$7.14779\text{x}10^2$
p=3	$3.83418\text{x}10^3$	$1.82604\text{x}10^3$	$9.75396\text{x}10^2$	$7.78767\text{x}10^2$
p=4	$3.76945\text{x}10^3$	$1.82241\text{x}10^3$	$1.01225\text{x}10^3$	$8.33994\text{x}10^2$
p=-1	$4.52956\text{x}10^3$	$2.21202\text{x}10^3$	$1.13612\text{x}10^3$	$8.30887\text{x}10^2$
p=-2	$4.43785\text{x}10^3$	$2.23572\text{x}10^3$	$1.22124\text{x}10^3$	$9.42685\text{x}10^2$
p=-3	$4.53546\text{x}10^3$	$2.31726\text{x}10^3$	$1.27029\text{x}10^3$	$9.88249\text{x}10^2$

CONCLUSIONS

In this study, we used the first order shear deformation theory (FSDT) coupled with the finite element method to maximize the frequency of simply supported laminated cylindrical shell for L/R ratios. We assume that a FG anisotropic shell in which the fiber orientations vary smoothly through shell thickness. It is seen that, for all L/R ratios the maximum frequency is obtained for p=-3 and as L/R ratio increases the fundamental frequency decreases.

REFERENCES

Zhi-yuan C., Hau-ying W., 2007, Free vibration of FGM cylindrical shells with holes under various boundary conditions, **Journal of Sound and Vibration**, Vol. 306, pages 227-237.

Haddadpor H., Mahmoudkhani S., Navazi H.M., 2007, Free vibration analysis of functionally graded cylindrical shells including thermal effects, **Thin-Walled Structures**, Vol. 45, pages 591-599.

Shakeri M., Akhlaghi M., Hoseini S.M., 2006, Vibration and radial wave propagation velocity in functionally graded thick hollow cylinder, **Composite Structures**, Vol. 76, pages 174-181.

Bhangale, R.K., Ganesan N, Padmanabhan C., 2006, Linear thermoelastic buckling and free vibration behavior of functionally graded truncated conical shells, **Journal of Sound and Vibration**, Vol. 292, pages 341-371.

Kadoli R., Ganesan, N., 2006, Buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature-specified boundary condition, **Journal of Sound and Vibration**, Vol. 289, pages 450-480.

Patel B.P., Gupta S.S., Loknath M.S., Kadu C.P., 2005, Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory, **Composite Structures**, Vol. 69, pages 259-270.