LATERAL STIFFNESS OF PYRAMID SHAPE BUILDINGS WITH INCLINED COLUMNS

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Key Words: pyramid shape building, inclined column, lateral stiffness, structural irregularity.

ABSTRACT: The effects of stiffness are very important therefore if the setting of the stiffening elements at structure and their geometrical specifications are not opted accurately, the structure may undergo amplify against the earthquake waves and the structure may be subject to fracture and may even lose its practical aspects.

If the stiffness of structure elements in multi-storey structures alters, it can precipitate the vibration moods of structure shape. Although noteworthy the definitions of lateral stiffness explain in a few codes but in many seismic codes don’t define.

Nevertheless high rise building with regular geometry have been increasing used in new seismic zone. On the other hand, in relevant areas, there are no specific criteria for such type of structural systems. One of the important behavior response specifications to pyramid structure is lateral stiffness; because of inclined column elements in this type of structure make these structural responses in different manner.

In pyramid structures reduction of stiffness in height don’t uniform and affects of inclined column in some stories, otherwise, this fact incertitude the regularity of pyramid building and this means equivalent lateral force procedure isn’t sufficient for design pyramid building and need to analysis with modal response spectrum or other appropriate and intricate analysis such as nonlinear static analysis or time-history for design these structures. Therefore in this paper numerous categories of pyramid buildings with different stories and slopes are selected, then analysis and design them. Afterward the result of irregularity of storey stiffness in all selected category, compare with limitation of horizontal and vertical structural irregularities of authenticable standards.

INTRODUCTION

Pyramid structures are those multi-storey building where the dimensions and the surface of the plan diminish from a specific storey at building in height where the pyramid shapes start it. A decrease in dimension plan of pyramid structure due to shapes of pyramid in one or two direction may substantially influence the behavior lateral stiffness behavior in symmetric and asymmetric structures. Unfavorable lateral stiffness behavior is one of the important causes of damage of tall buildings during strong ground motion especially in seismic zones.

The figure perused in this article is an option for utilization of diagonal columns to diminish the plan level in question. Considering the oblique nature of the diagonal members of the pyramid structures have been portended about the dynamic performance and one of the most important and effective response of variation lateral stiffness from the below story to upper one.
This study for investigate the lateral stiffness divided in two parts. Part one includes of selected numerous categories of pyramid buildings with different stories and slope. This part have two categories buildings, each category have three type buildings; symmetric pyramid, asymmetric pyramid and regular structures which in 8-Storey & 16-Storey were employed as test examples. Symmetric variants of the buildings were design according to AISC seismic provision of steel structure [1] pre-standards. Intentionally, asymmetric was not considered in the design in order to eliminate any tensional code influence from the result.

In part two; symmetric pyramid 8-storey and 16-storey structures with different slope faces which changes from 200% to 900% at side faces are selected and compare lateral stiffness of test structures with regular structures with same storey and shape. In this paper some of the main results of the study are presented which mainly in terms of lateral stiffness and afterward the conclusion of this study describe.

**THEORY and METHOD**

The effects of the relative stiffness of the upper stories and the establishment of soft stories in the underneath of the structure may be causes to analysis and design the pyramid structures as irregular rules according to authenticable standards. For compare the lateral stiffness of these structures, first we need to describe the method of ASCE7-05 [2], IBC 2000 [3] and Japan’s code standards [4]. According to ASCE7-05 [2] two type of irregularity may be occurring to structure systems.

a) Stiffness–Soft storey irregularity
b) Stiffness-Extreme Soft-storey irregularity.

c) Stiffness-Soft storey irregularity is defined to exist where there is a storey in which the lateral stiffness is less than 70% of that storey above or less than 80% of the average stiffness of the three stories above. Stiffness-Extreme Soft storey irregularity is defined to exist where there is storey in which the lateral stiffness is less than 60% of that in the storey above or less than 70% of the average stiffness of three stories above.

By the same idea, Japan’s codes have same regulation for irregularity of lateral stiffness. But the momentous point that “how we can determine the lateral stiffness of each – storey in structure?”

This point doesn’t mention and describe in many authorized standards. For example the important codes, UBC, IBC, NBCE, and NEHRP don’t demonstrate any method for determination of lateral stiffness storey. One of the methods that presented lateral stiffness is from earthquake institute of japans’ [5].

According to this article stiffness of storey figured out to divide designed earthquake shears to the value of variation displacement pertinent of above storey and below one see in Equation (1).

\[ K_i = \frac{V_i}{\text{Displ.}(i - r)} \]  

(1)

\( i = \text{number of storey}, \ V_i = \text{shear force of storey} \ i, \ D_i = \text{displacement at storey} \ i, \ D_r = \text{displacement at storey} \ r, \ K_i = \text{lateral stiffness} \)

In order to recognize the comparison behavior of pyramid structure to regular structure, this study divided in two parts:
Part one discusses about the differences between symmetric and asymmetric pyramid structure to regular structure in 8-storey and 16-storey structures with constant slope faces for pyramid structure with 60% for asymmetric pyramid structure and 200% for symmetric pyramid slope faces.

Part two discusses about lateral stiffness behavior in symmetric pyramid shape structure with different slope which slopes of these structures changes from 200% to 900% in exterior sides faces at symmetric pyramid 8-storey and 16-storey structures.

Examples (Test Structures):

**Part one:** Six-storey steel frame pyramid buildings are used as the test structures. Three different structural style in two level heights as 8-storey & 16-storey are employed. [Symmetric and asymmetric pyramid building compare to regular structure.] (Regular structure denoted a structure in which the dimensions of the plan surface don’t change in height and vary in uniform manner).

3D of structures, schematic plans and elevations of the building are shown in Figure-1 to Figure-2 for plan configuration and for 3D shape in Figure-3 to Figure-8. The storey heights are 3.3(m) for all storeys. The dead and live loads of 600 kg/m² and 200 kg/m², respectively, were used for gravity loads, and the earthquake design base shear was determined base on IBC 2000 [3]. Response modification factor = 8.0 for special steel moment frame was used. A36 steel was used for every structural member. The structural design was carried out using the program ETABS version 8.50 [6]. To take the conventional design practice into consideration, the same structural members were used in three consecutive stories.

The Symmetric, Asymmetric & Regular structures were designed to standards AISC, seismic provision of steel structure [1] and ASCE7-05 [2]. The connections were moment-resistant frames and design spectrum for stiff soil type II, normalized to peak ground acceleration of 0.35g, was used. A conservation estimate of the natural period was made, which resulted in base shear about 10% of the total weight the building. The main difference between the three systems lies in their geometrical buildings in height. (For both in directions X and Y)
The initial vertical loads are distributed in all floors with deck system. However the influence of these differences of assigning method of initial loads in the X and Y direction, respectively, on the seismic response and lateral stiffness is very small. The shapes of pyramid building (Symmetric or Asymmetric) are:

8-storey Symmetric pyramid structure is with slope of 200% from 4 sides which commence from the level of the 5th floor and will continue up to the roof level. (Figure-3)

8-storey pyramid asymmetric structure is with slope of 60% that exist from 2 sides which commence from the level 5th and will continue up to roof level floor. (Figure-4)

8-storey regular structure is without making any alternation in plan dimensions regarding the altitude. (Figure-5)

16-storey Symmetric pyramid structure is with slope of 200% from 4 sides which commence from the level of the 10th floor and will continue up to the roof level. (Figure-6)

16-storey pyramid asymmetric structure is with slope of 60% that exist from 2 sides which commence from the level 10th and will continue up to roof level floor. (Figure-7)

16-storey regular structure is without making any alternation in plan dimensions regarding the altitude. (Figure-8)

The 3D shapes of test buildings part one are as below figures:

For investigate the lateral stiffness of test structure according to Equation (1), carried out and then scaled the values with first floor at each structure. In Table-1 and Table-2 the value of relative lateral stiffness of 8-storey and 16-storey test structure are shown, respectively.
Table-1: Scaled lateral stiffness of 8-storey test structures. (Sym, asym & regular)

<table>
<thead>
<tr>
<th>8th Floor (Kr)</th>
<th>7th Floor (Kr)</th>
<th>6th Floor (Kr)</th>
<th>5th Floor (Kr)</th>
<th>4th Floor (Kr)</th>
<th>3rd Floor (Kr)</th>
<th>2nd Floor (Kr)</th>
<th>1st Floor (Kr)</th>
<th>8_Storey Building</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.158</td>
<td>3.55</td>
<td>1.261</td>
<td>0.511</td>
<td>0.513</td>
<td>0.569</td>
<td>0.614</td>
<td>1</td>
<td>Symmetric</td>
</tr>
<tr>
<td>1.451</td>
<td>0.741</td>
<td>1.054</td>
<td>0.565</td>
<td>0.531</td>
<td>0.573</td>
<td>0.613</td>
<td>1</td>
<td>Asymmetric</td>
</tr>
<tr>
<td>0.184</td>
<td>0.356</td>
<td>0.424</td>
<td>0.505</td>
<td>0.527</td>
<td>0.599</td>
<td>0.643</td>
<td>1</td>
<td>Regular</td>
</tr>
</tbody>
</table>

Table-2: scaled lateral stiffness of 16-storey test structures (sym, asym & regular)

<table>
<thead>
<tr>
<th>16th FL. (Kr)</th>
<th>15th FL. (Kr)</th>
<th>14th FL. (Kr)</th>
<th>13th FL. (Kr)</th>
<th>12th FL. (Kr)</th>
<th>11th FL. (Kr)</th>
<th>10th FL. (Kr)</th>
<th>9th FL. (Kr)</th>
<th>8th FL. (Kr)</th>
<th>7th FL. (Kr)</th>
<th>6th FL. (Kr)</th>
<th>5th FL. (Kr)</th>
<th>4th FL. (Kr)</th>
<th>3rd FL. (Kr)</th>
<th>2nd FL. (Kr)</th>
<th>1st FL. (Kr)</th>
<th>16_St BL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.57</td>
<td>0.19</td>
<td>0.38</td>
<td>0.28</td>
<td>0.27</td>
<td>0.26</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.32</td>
<td>0.34</td>
<td>0.36</td>
<td>0.41</td>
<td>0.53</td>
<td>1</td>
<td>Sym</td>
</tr>
<tr>
<td>0.62</td>
<td>0.27</td>
<td>0.18</td>
<td>0.25</td>
<td>0.23</td>
<td>0.26</td>
<td>0.26</td>
<td>0.28</td>
<td>0.3</td>
<td>0.33</td>
<td>0.35</td>
<td>0.39</td>
<td>0.44</td>
<td>0.47</td>
<td>0.52</td>
<td>1</td>
<td>Asym</td>
</tr>
<tr>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.14</td>
<td>0.16</td>
<td>0.17</td>
<td>0.22</td>
<td>0.25</td>
<td>0.26</td>
<td>0.28</td>
<td>0.30</td>
<td>0.32</td>
<td>0.37</td>
<td>0.42</td>
<td>0.52</td>
<td>1</td>
<td>Reg</td>
</tr>
</tbody>
</table>

Note: The value scale and normalized to first storey (1th Floor).

Kr: Lateral Stiffness which normalized at storey “1”.

The lateral stiffness variability of 8-storey and 16-storey structures and variation of scaled lateral stiffness are shown in Figure-9 and Figure-10.

Figure-9: Kr in 8-Storey Structures

Figure-10: Kr in 16-Storey Structures

By attention to above Tables and Figures, it’s express the relative lateral stiffness in upper storeys in pyramid shape structure increased suddenly (roof and below roof floor). Furthermore this increase for both 8-storey and 16-storey in symmetric pyramid shape is more major than asymmetric structure shape.

For comparison the lateral stiffness with standards requirement limitation, the difference between each adjacent lateral stiffness storeys presented to percent(%) in Table-3 & Table-4 and specified with parameter δ in Equation (2):
\[ \delta_i = \{(K_r - K_{r+1})/K_r\} \]

where:
- \( i \): number of storey
- \( K_r \): relative lateral stiffness
- \( \delta_i \): the difference between each adjacent storeys

**Table -3:** parameter \( \delta \), in test structures 8-storey buildings

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>95%</th>
<th>181%</th>
<th>147%</th>
<th>0.4%</th>
<th>10%</th>
<th>8%</th>
<th>38%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sym.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>96%</th>
<th>29%</th>
<th>86%</th>
<th>6%</th>
<th>7%</th>
<th>6%</th>
<th>48%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asym.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>48%</th>
<th>16%</th>
<th>16%</th>
<th>4%</th>
<th>12%</th>
<th>7%</th>
<th>36%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

By comparisons between the values in Table-3, we could express that in 8-storeys test structures:

- a) In symmetric pyramid structure, the lateral stiffness in storey “5” to storey “6” and storey “6” to storey “7” increase suddenly, as, 147% (one hundreds forty seven) and 181% (one hundreds eighty one).
- b) In asymmetric pyramid structures, the lateral stiffness from storey “5” to storey “6” and storey “7” to storey “8” increased suddenly, as, 86% (eighty six percent) and 96% (ninety six percent) in storey “7” and storey “8”, respectively.

The result in test cases 8-storey buildings express that stiffness–Extreme Soft storey Irregularity occurred according to ASCE 7-05 [2]. For example, lateral stiffness in storey “5” in symmetric and asymmetric pyramid 8-storey less than 60% of that storey above. [Refer to ASCE7-05 code]

Similar behavior can be observed in the case of 16-storey buildings. However, some influence as same as 8-storey cases can be noticed in Table-4.

**Table -4:** parameter \( \delta \), in test structures 16-storey buildings

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>91%</th>
<th>200%</th>
<th>50%</th>
<th>30%</th>
<th>10%</th>
<th>10%</th>
<th>10%</th>
<th>0%</th>
<th>0%</th>
<th>25%</th>
<th>6%</th>
<th>5%</th>
<th>12%</th>
<th>22%</th>
<th>47%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sym.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| \( \delta \) | 130% | 50% | 28% | 8% | 11% | 0% | 7% | 6% | 0.09 | 6% | 0% | 10% | 11% | 6% | 10% |
|-------------|------|-----|-----|----|-----|----|----|----|------|----|----|------|------|----|----|----|
| Asym.       |     |     |     |    |     |    |    |    |      |    |    |      |      |    |    |    |

| \( \delta \) | 12% | 11% | 35% | 12% | 6% | 22% | 12% | 4% | 0.07 | 6% | 6% | 13% | 12% | 19% | 48% |
|-------------|-----|-----|-----|-----|----|-----|-----|----|------|----|----|------|------|----|----|----|
| Reg.        |     |     |     |     |    |     |     |    |      |    |    |      |      |    |    |    |

The result for test case 16-storey building are as same as 8-storey and indicated that stiffness extreme soft storey irregularity occur in pyramid 16-storey structures.

**Part two**

Part two discusses about lateral stiffness behavior in symmetric pyramid shape structure with different slope which slope of these structures changes from 200% to 900% in exterior sides faces at symmetric pyramid 8-storey and 16-storey structures.
All of the structure specification and methods of modeling as same as part one. The aim of this part is to demonstrate the effect of different slopes of inclined element in side faces in lateral stiffness of structure and comparison the lateral stiffness of these structures with regular structure. The elevation of model structures with slopes 200% to 900% are shown as Figure-11 for 8-storey structures and Figure-12 for 16-storey structures.

In 8-storey structures the slope start at level 4th and in 16-storey structure slope start at level 10th. The meaning of slope 200% means 2 in height and 1 in horizontal. Lateral stiffness determined according to Equation (1).

In Figure-13 and Figure-14, curves of relative lateral stiffness of symmetric pyramid 8-storey and 16-storey structure with 200% to 900% slopes are shown. All lateral stiffness scaled with lateral stiffness of first floor.

![Figure-11: Configuration of 8-storey Model structures](image)

![Figure-12: Configuration of 16-storey Model structures](image)

![Lateral Stiffness of Symmetric Pyramid 8-Storey Structure](image)

**Figure-13**: Kr in 8-Storey Pyramid Symmetric Structures from 200% to 900% slopes
By investigating the Figure-13 & Figure-14, we found that in 8-storey structures at slopes 200%, 300% and 400% have major effect on lateral stiffness structures due to inclined element at side face of buildings and at further slopes (500%, 600%, 700%, 800% and 900%) the effect of inclined element diminished and lateral stiffness curve get near to regular structures curve, also in 16-storey structures at slope 200% and 300% lateral stiffness have variation in upper storey and in further slopes the effect of inclined element decreased. But the effect of inclined element causes that lateral stiffness curves in pyramid structure with any slopes have major value compared to regular structure.

**CONCLUSIONS**

Multi-storey and high rise buildings have been increasing in new seismic zones. Design and application of symmetric structures have been recommended in most reference codes to resist earthquakes, and any deviation of the geometrical structural system demands complicated precise analyses to determine the seismic structural response. The pyramid shape structures are one of the most applicable shapes that are used for designing of high rise buildings. The structural systems in pyramid buildings are mostly moment frames which are usually composed of beams and vertical columns in middle part and inclined columns in side faces. These structures by geometrical area may be constructed in asymmetric and symmetric shapes. The inclined elements due to seismic forces have such effects in lateral stiffness behavior of structure. The response of symmetric and asymmetric pyramid structures with inclined element due to lateral stiffness effects have some result that listed in below, however it is encouraging that the majority of trends and conclusions are consistent with those obtained in selected test structures but can get some result from this research.

Based on the results of this study the following conclusions relevant to comparison the asymmetric pyramid and symmetric pyramid and regular test structures due to elastic analysis can be drawn:
• The lateral stiffness in symmetric and asymmetric pyramid building has much further value than regular structures.

• The relative lateral stiffness between each adjacent storey exceeded from seismic codes requirement. (According to ASCE 7-05 [2])

• Because of exceed limitation of relative lateral stiffness refer to seismic codes ASCE 7-05 [2], these structures have irregularity in structure systems because lateral stiffness is more than 70% of that storey above therefore equivalent lateral force procedure isn’t sufficient for design pyramid building and need to analysis with modal response spectrum or other appropriate and intricate analysis.

• The symmetric pyramid structure with soft slope (500%,600%,...) due to inclined element in side faces has much better behavior than regular structure due to seismic loads.

In all cases, the examples in this study demonstrate that, symmetric and asymmetric pyramid structures especially with extreme slope such as 200% and 300%, due to seismic forces has not equal variation of lateral stiffness in each storey’s as same as regular buildings and need precise analysis such as modal response spectrum, non-linear static analysis or Time-history analysis to design it in safe manner.

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